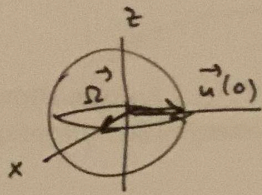


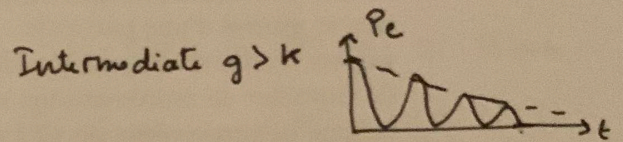
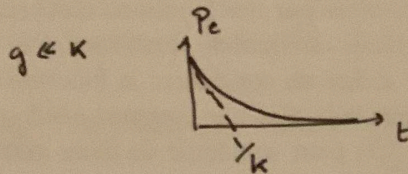
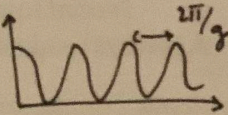
I 1-



Rotation in (yz) plane

When prepared in $|0\rangle + |1\rangle$, $\vec{u}$ is aligned along (ox)
 $\Rightarrow$ no rotation.

2. a) $g \gg \kappa$ Rabi oscillation P_e



b) Markovian bath $\Rightarrow$ exponential decay - hence valid for $g \ll \kappa$.
 In the other case ($g > \kappa$), the photon "oscillates" between cavity and atom: no loss of memory

c) $\tau_c \sim 1/\kappa$, i.e. related to the width of the continuum, here κ .

3. a) $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1$ yes -

b) If product $\Rightarrow (\alpha|0\rangle + \beta|1\rangle) \otimes (\gamma|0\rangle + \delta|1\rangle) = |\psi\rangle$

$$\Rightarrow \alpha\gamma = \frac{1}{2} = \alpha\delta = \beta\gamma \quad \text{and} \quad \beta\delta = \frac{1}{2} e^{i\phi}$$

$$\Rightarrow \alpha\beta\gamma\delta = \frac{1}{4} \quad \text{and} \quad \alpha\gamma\beta\delta = \frac{1}{4} e^{i\phi} \quad \text{OK if } e^{i\phi} = 1 \Rightarrow \phi = 2\pi p.$$

As soon as $\phi \neq 0(2\pi) \Rightarrow$ entangled

$$c) \quad \rho_A = \langle 0 | \psi \psi^\dagger | 0 \rangle_B + \langle 1 | \psi \psi^\dagger | 1 \rangle_B$$

$$= \frac{1}{4} (|0\rangle + |1\rangle)(\langle 0| + \langle 1|) + \frac{1}{4} (|0\rangle + e^{i\phi}|1\rangle)(\langle 0| + e^{-i\phi}\langle 1|)$$

$$= \frac{1}{2} |0\rangle\langle 0| + \frac{1}{2} |1\rangle\langle 1| + \frac{1}{4} (1 + e^{i\phi}) |0\rangle\langle 1| + \frac{1}{4} (1 + e^{-i\phi}) |1\rangle\langle 0|$$

$$\rho_A = \frac{1}{2} \begin{pmatrix} 1 & \frac{1}{2}(1 + e^{i\phi}) \\ \frac{1}{2}(1 + e^{-i\phi}) & 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & e^{i\phi/2} \cos \phi/2 \\ e^{-i\phi/2} \cos \phi/2 & 1 \end{pmatrix}$$

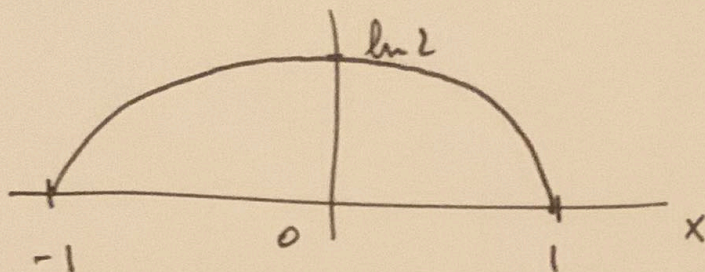
d) Eigenvalues of ρ_A $(1-x)^2 - \cos^2 \phi/2 = 0$

$$\lambda = \frac{1}{2} [1 \pm \cos \phi/2]$$

Hence:

$$S_A = - \sum \lambda_i \ln \lambda_i = - \frac{1}{2} (1 + \cos \phi/2) \ln \left[\frac{1}{2} (1 + \cos \phi/2) \right] - \frac{1}{2} (1 - \cos \phi/2) \ln \left[\frac{1}{2} (1 - \cos \phi/2) \right]$$

$$\text{or } S_A = + \ln 2 - \frac{1}{2} \left[(1+x) \ln(1+x) + (1-x) \ln(1-x) \right]$$



When $x = \cos \phi/2 = 0$

The state $|\psi\rangle$ is maximally entangled, i.e. for $\phi = \pi$

4- b) Density matrix of 2 qbits $\rightarrow$ 16 coefficients to measure.

Each require 500 repetitions $\rightarrow 16 \times 500$ meas^t.

Exp running at 10 Hz $\Rightarrow T = 16 \times 500 / 10 = 800 \text{ sec.} \approx 15 \text{ min.}$

a) Apply rotations on each qbit to measure $\langle \sigma_j \otimes \sigma_k \rangle$

II 5.

$$\sigma_{\theta} = \begin{pmatrix} 0 & e^{-i\theta} \\ e^{i\theta} & 0 \end{pmatrix}$$

$$\text{Hence } \langle \sigma_{\theta_A} \otimes \sigma_{\theta_B} \rangle = \frac{1}{2} (\langle 00 | - \langle 10 |) (e^{i\theta_A} |1\rangle e^{-i\theta_B} |0\rangle - e^{-i\theta_A} e^{i\theta_B} |01\rangle) \\ = -\cos[(\theta_A - \theta_B)]$$

6. The ~~disturbance~~ imperfect detection means that with a probability η one correctly measures the state $|\psi\rangle$, while with a probability $1-\eta$, one gets a totally random result.

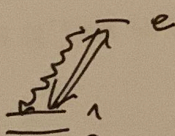
Hence ρ is a statistical mixture of the 2 cases:

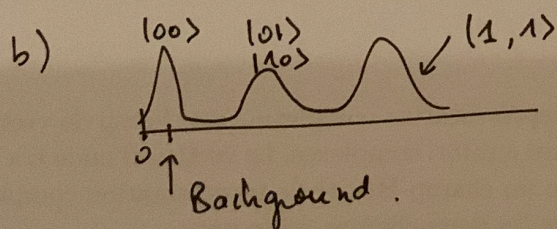
$$\rho = \eta |\psi\rangle\langle\psi| + (1-\eta) \rho'$$

$$7. \quad E(\theta_A \theta_B) = \eta \text{Tr} [|\psi\rangle\langle\psi| \sigma_{\theta_A} \otimes \sigma_{\theta_B}] + (1-\eta) \underbrace{\text{Tr} [\rho' (\sigma_{\theta_A} \otimes \sigma_{\theta_B})]}_{\text{uncorrelated state}} \\ = -\eta \cos[(\theta_A - \theta_B)]$$

$$8. \quad S_Q^{\max} = 2\sqrt{2} \eta$$

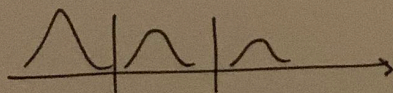
9. you need $\eta > \frac{2}{2\sqrt{2}}$ to violate Bell $\Rightarrow \eta \gtrsim 0.7$.

10. a)  laser tuned on $1 \leftrightarrow 0$ transition.
Collect fluorescence $\Rightarrow$ atom in $|1\rangle$
no light $\Rightarrow$ atom in $|0\rangle$



c) For one atom (state $|01\rangle$ or $|10\rangle$) collect about 60 photons in 20 sec $\Rightarrow 3 \text{ ph/sec}$.

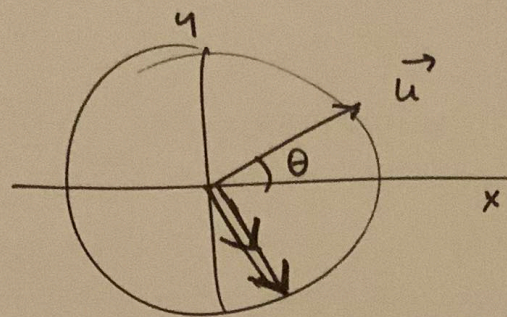
d) the 3 histograms are well separated: a threshold between $|00\rangle$ and $|10\rangle$ and $|10\rangle - |11\rangle$ allows one to distinguish between the 2



- 11 Apply rotation on qubit state, for example with microwave coupling the state A and B. Other option: a Raman transition, which allows one to perform independent rotation on each atom.

For the rotation operation: $\sigma_\theta = \sigma_x \cos \theta + \sigma_y \sin \theta$,

the angle.



One has to apply a $\pi/2$ -pulse around the axis $\perp \vec{u}$ to read out the state in the z basis.

12 $\langle S_{exp} \rangle \approx 2,25$

dispersion $\left(0,03^2 + 0,025^2 + 0,01^2 + 0,006^2 + 0,005^2 \right)^{1/2} \approx 0,03$

Violation $\frac{2,25 - 2}{0,03} = \frac{0,25}{0,03} \approx 80$

13 $2\sqrt{2} \eta = 2,25 \Rightarrow \eta = 0,79.$

14 Atoms too close to ~~be separated by a~~ to test the non-locality.