

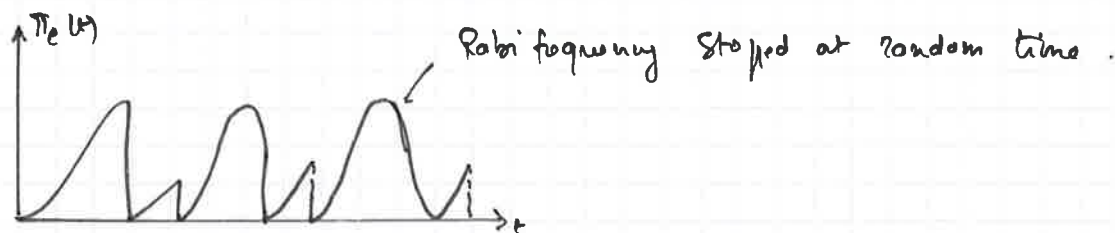
I 1. Measure in 2 different basis, eg $|0\rangle |1\rangle$ and $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$
 for $\alpha|0\rangle + \beta|1\rangle$ $P_0 = |\alpha|^2$ $P_+ = |\langle +|\psi\rangle|^2 = \left| \frac{1}{\sqrt{2}} (1 \ 1) \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \right|^2 = \frac{1}{2} |\alpha + \beta|^2$
 $P_- = |\beta|^2$ $P_- = \frac{1}{2} |\alpha - \beta|^2$

For $\rho = \begin{pmatrix} |\alpha|^2 & 0 \\ 0 & |\beta|^2 \end{pmatrix}$ $P_0 = |\alpha|^2$ and $P_1 = |\beta|^2$, but $P_+ = \text{Tr}(\rho |X+I\rangle)$

As $|X+I\rangle = \frac{1}{2} (|0\rangle + |1\rangle)(\langle 0| + \langle 1|) = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$\text{Tr}[\rho |X+I\rangle] = \text{Tr}\left[\frac{1}{2} \begin{pmatrix} |\alpha|^2 & 0 \\ 0 & |\beta|^2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\right] = \frac{1}{2} (|\alpha|^2 + |\beta|^2) = \frac{1}{2} \neq P_+.$

2.



3. Application of continuum of variable width. As $g \ll \kappa$, the continuum is broad $\Rightarrow T_c \sim 1/\kappa \approx \frac{1}{2\pi \times 10^9} \sim 10^{-10} \text{ s}$

4. $|\psi_{SE}\rangle = \frac{1}{\sqrt{2}} (|0\rangle |+\rangle + |1\rangle |-\rangle)$

• $\| |\psi_{SE}\rangle \|^2 = \frac{1}{2} + \frac{1}{2} = 1$

• It is not a product state: can not be written $(\alpha|0\rangle + \beta|1\rangle)(\gamma|+\rangle + \delta|-\rangle)$

• By a density matrix: $\rho_S = \text{Tr}_E |\psi_{SE}\rangle \langle \psi_{SE}|$, where we use the $|+\rangle |-\rangle$ basis for E

$\rho_S = \frac{1}{2} \left(\langle +|(0\rangle\langle +| + |1\rangle\langle -|) \right) \left(\langle 0|\langle +| + \langle 1|\langle -| \right) |+\rangle + \langle -| \quad |-\rangle$
 $= \frac{1}{2} (|0\rangle\langle 0| + |1\rangle\langle 1|)$

II 5. Rabi oscillation at a frequency Ω

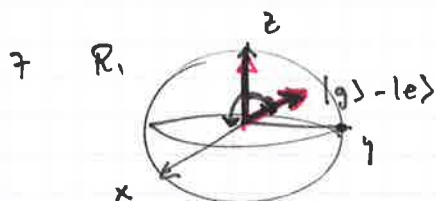
$$|g\rangle \rightarrow \cos \frac{\Omega t}{2} |g\rangle - i e^{-i\phi} \sin \frac{\Omega t}{2} |e\rangle$$

where ϕ is the phase of the μ w field. Choosing $\phi = -\pi/2$ and $\frac{\Omega t}{2} = \frac{\pi}{4}$ leads to $|g\rangle \rightarrow \frac{1}{\sqrt{2}}(|g\rangle - e^{-i\phi} |e\rangle)$, where $|e\rangle \rightarrow \frac{1}{\sqrt{2}}(e^{i\phi} |g\rangle + |e\rangle)$

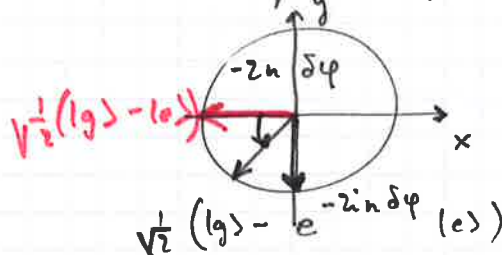
The associated rotation such that $|\psi'\rangle = R(\phi) |\psi\rangle$ is thus:

$$R(\phi) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{i\phi} \\ -e^{-i\phi} & 1 \end{pmatrix}$$

$$6. |g\rangle \xrightarrow{R_1} \frac{1}{\sqrt{2}}(|g\rangle - e^{-i\phi} |e\rangle) = \frac{1}{\sqrt{2}}(|g\rangle - |e\rangle) \xrightarrow{C} \frac{1}{\sqrt{2}}(e^{in\delta\phi} |g\rangle - e^{-in\delta\phi} |e\rangle)$$



C: In the equatorial plane: Rotation by $2n\delta\phi$



$$8. \frac{1}{\sqrt{2}} (e^{in\delta\phi} |g\rangle - e^{-in\delta\phi} |e\rangle) \rightarrow \frac{1}{2} [e^{in\delta\phi} (|g\rangle - e^{i\phi} |e\rangle) + e^{-in\delta\phi} (|g\rangle + e^{i\phi} |e\rangle)]$$

$$\Rightarrow P_{g\phi}(n) = \left| \frac{1}{2} (e^{in\delta\phi} + e^{-in\delta\phi} e^{i\phi}) \right|^2$$

$$= \frac{1}{2} [1 + \cos(2n\delta\phi - \phi)]$$

$$P_{e\phi}(n) = \left| \frac{1}{2} (-e^{in\delta\phi - i\phi} + e^{-in\delta\phi}) \right|^2$$

$$= \frac{1}{2} [1 - \cos(2n\delta\phi - \phi)]$$

With $g \leftrightarrow 0$
 $e \leftrightarrow 1$

$$P(j\phi|n) = \frac{1}{2} [1 + \cos(2n\delta\phi - \phi + j\pi)]$$

9. To follow the trajectory, you must not destroy the photons

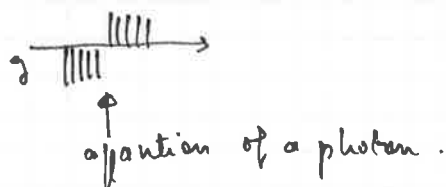
$\Rightarrow$ QND (on top of that should not affect the quantum state)

10 $\phi=0 \Rightarrow P_e(n \neq 1) = \frac{1}{2} (1 - \cos 2\delta\phi)$ for $n=1$

$P(g|n=1) = \frac{1}{2} (1 + \cos 2\delta\phi)$

If $2\delta\phi = \pi$, then $P_e(n=1) = 1$ and $P_g(n=1) = 0$.

- 11- When no photon is in the cavity the atom should exit R_2 in g .
As soon as $n=1$, the atoms come out in e .



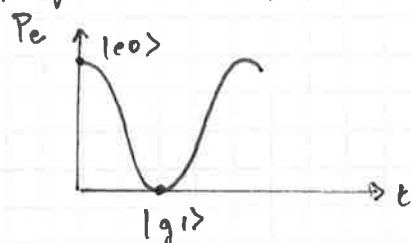
When the photon leaves the cavity, the atoms again exit in g .

- 12- • Thermal excitation of the field $\rightarrow$ creation of the photon.

• $\bar{n} = \frac{1}{e^{\frac{\hbar\omega_0}{k_B T}} - 1}$

Here. $\frac{\hbar\omega_0}{k_B T} = \frac{6,6 \times 10^{-34} \times 51 \times 10^9}{1,4 \times 10^{-23} \times 0,8} = 3 \Rightarrow \bar{n} = \frac{1}{e^3 - 1} \approx 0,05$

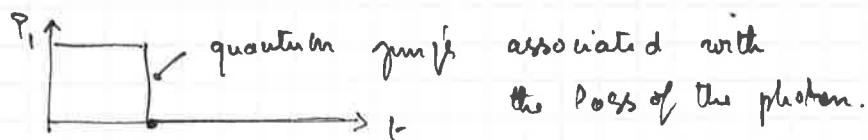
- 13 The atom - C is coupled. From lecture 2, means $|e0\rangle \rightarrow |g1\rangle$ if one adjust the cavity-atom interaction time. (for example varying the velocity of the atoms). For a π "pulse" $|e0\rangle \rightarrow |g1\rangle$



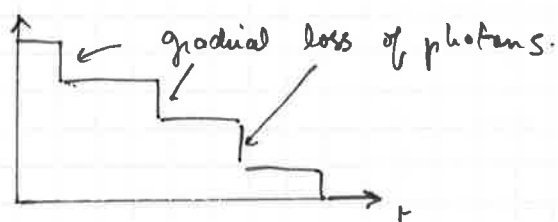
and 1 photon is created in the cavity.

- 14- Fig 26. Shows the probab. to have 1 photon in the cavity

For a single trajectory



Several trajectories



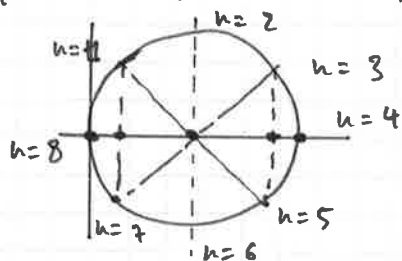
15 Averaging over many trajectories, we should recover the cavity decay time: $P_c(t) \sim e^{-t/T_c}$. Looking in Fig 2(b)-c, we find $T_c \approx 0,5 \text{ sec}$ for the γ_c decay time.

Hence $D = c T_c = 3 \times 10^8 \times 0,5 \approx 0,5 \times 10^8 \text{ m} \approx 50\,000 \text{ km}$.

round trips: $\frac{5 \times 10^7}{2,7 \times 2 \times 10^{-2}} \approx 9 \times 10^8$.

16.

for $q=4$ and $\phi=0$ $P(c|n) = \frac{1}{2} (1 - \cos n\pi/4)$



$> 2q$ photons are not distinguishable because the function $P(en)$ is modulo $2q$.

17.

Bayes Rule: $P(n|e\phi) \cdot P(e\phi) = P_0(n) P(e\phi|n)$

Hence $P(n|e\phi) \propto P_0(n) P(e\phi|n)$

18 $P(n|e\phi) \propto P_0(n) \frac{1}{2} (1 - \cos(\frac{n\pi}{2q} - p\pi/q)) \propto P_0(n) [1 - \cos((n-p)\frac{\pi}{q})]$

If one measures an atom in e , then $n \neq p$ otherwise you would have $P_e = 0$. Hence, by normalization of the proba, you must have more proba to find $n \neq p$ photons.

19. $P_N(n) \propto \prod_{k=1}^N P_0(n) P(j(k)\phi(k)|n)$ as the detection of each of the N atom modifies the proba distribution.

20

The figure shows that by properly choosing the phase, one can steer the probability of find photon towards a unique value, i.e. prepare a Fock state. Here we observe that $n_{\text{find}} = 7$.

21 $H = \frac{\hbar \Omega}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Rightarrow E_{\pm} = \pm \frac{\hbar \Omega}{2}$ and $| \pm \rangle = \frac{1}{\sqrt{2}} (| 0 \rangle \pm | 1 \rangle)$ 5

22 $\frac{d\rho}{dt} = -i \frac{\Omega}{2} (\sigma_x \rho - \rho \sigma_x)$ where $\sigma_x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ in the $| \pm \rangle$ basis

$$= -i \frac{\Omega}{2} \left[\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \rho_{++} & \rho_{+-} \\ \rho_{-+} & \rho_{--} \end{pmatrix} - \begin{pmatrix} \rho_{++} & \rho_{+-} \\ \rho_{-+} & \rho_{--} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right]$$

$$= -i \frac{\Omega}{2} \begin{pmatrix} 0 & 2\rho_{+-} \\ -2\rho_{-+} & 0 \end{pmatrix}$$

$$\Rightarrow \dot{\rho}_{++} = \dot{\rho}_{--} = 0 \quad \text{and} \quad \rho_{\pm\mp} = \mp i \Omega \rho_{\pm\mp}$$

$$\Rightarrow \rho_{++}(t) = \rho_{++}(0) \quad \text{and} \quad \rho_{--}(t) = \rho_{--}(0)$$

$$\rho_{+}(t) = \rho_{+}(0) e^{-i\Omega t}; \quad \rho_{-}(t) = \rho_{-}(0) e^{i\Omega t}$$

23 $\rho_{11}(t) = \langle 1 | \rho | 1 \rangle = \frac{1}{2} (\langle + | - \langle - |) \rho (| + \rangle - | - \rangle)$, as $| 1 \rangle = \frac{1}{\sqrt{2}} (| + \rangle - | - \rangle)$

$$= \frac{1}{2} [\rho_{++}(t) + \rho_{--}(t) - \rho_{+-}(t) - \rho_{-+}(t)]$$

$$= \frac{1}{2} \left[\underbrace{\rho_{++}(0) + \rho_{--}(0)}_{=1} - \rho_{+-}(0) e^{-i\Omega t} - \rho_{-+}(0) e^{i\Omega t} \right]$$

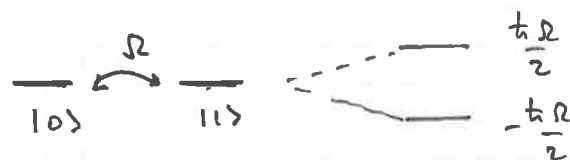
24 $\rho_{11}(0) = 0; \quad \rho_{00}(0) = 1$

As $| 0 \rangle = \frac{1}{\sqrt{2}} (| + \rangle + | - \rangle) \Rightarrow \rho = | 0 \rangle \langle 0 | = \frac{1}{2} (| + \rangle \langle + | + | - \rangle \langle - | + | + \rangle \langle - | + | - \rangle \langle + |)$

$$\Rightarrow \rho_{+-}(0) = \rho_{-+}(0) = \frac{1}{2}$$

And $\rho_{11}(t) = \frac{1}{2} (1 - \cos \Omega t)$

25



Preparing the system in $|0\rangle \Rightarrow$ oscillation between $|0\rangle$ and $|1\rangle$, analogous to a double well potential (eg oscillation in $\text{NH}_3 \dots$)

26. $L_{\pm}$ represent quantum jumps from $0 \xrightarrow{L_+} 1$
 $1 \xrightarrow{L_-} 0$

$\Gamma_{\pm}$ rate of jump: $\Gamma_{\pm} dt$ is the probab to undergo a jump during dt .

$$\begin{aligned} 27 \quad \frac{d\rho}{dt} &= \sum_{\alpha} L_{\alpha} \rho L_{\alpha}^{\dagger} - \frac{1}{2} L_{\alpha}^{\dagger} L_{\alpha} \rho - \frac{1}{2} \rho L_{\alpha}^{\dagger} L_{\alpha} \\ &= \Gamma_+ (\sigma_+ \rho \sigma_- - \frac{1}{2} \sigma_- \sigma_+ \rho - \frac{1}{2} \rho \sigma_- \sigma_+) \\ &\quad + \Gamma_- (\sigma_- \rho \sigma_+ - \frac{1}{2} \sigma_+ \sigma_- \rho - \frac{1}{2} \rho \sigma_+ \sigma_-) \end{aligned}$$

Recall $\sigma_+ = |1\rangle\langle 0|$; $\sigma_- = |0\rangle\langle 1|$

$$\begin{aligned} \text{hence } \frac{d\rho}{dt} &= \Gamma_+ (|1\rangle\langle 0| \rho_{00} - \frac{1}{2} |0\rangle\langle 0| \rho - \frac{1}{2} \rho |0\rangle\langle 0|) \\ &\quad + \Gamma_- (|0\rangle\langle 1| \rho_{11} - \frac{1}{2} |1\rangle\langle 1| \rho - \frac{1}{2} \rho |1\rangle\langle 1|) \end{aligned}$$

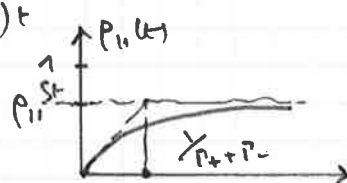
$$\begin{aligned} \Rightarrow \dot{\rho}_{00} &= -\Gamma_+ \rho_{00} + \Gamma_- \rho_{11} \quad \text{and } \dot{\rho}_{00} + \dot{\rho}_{11} = 0 \text{ as it should!} \\ \dot{\rho}_{11} &= +\Gamma_+ \rho_{00} - \Gamma_- \rho_{11} \end{aligned}$$

$$28. \text{ As } \rho_{00} + \rho_{11} = 1 \Rightarrow \dot{\rho}_{00} = 0 = -\Gamma_+ \rho_{00} + \Gamma_- (1 - \rho_{00})$$

$$\Rightarrow \rho_{00}^{st} = \frac{\Gamma_-}{\Gamma_- + \Gamma_+} \quad \text{and} \quad \rho_{11}^{st} = \frac{\Gamma_+}{\Gamma_- + \Gamma_+}$$

$$29 \quad \rho_{00}(t) = \rho_{00}^{st} + A e^{-(\Gamma_+ + \Gamma_-)t}; \quad \rho_{11}(t) = \rho_{11}^{st} + B e^{-(\Gamma_+ + \Gamma_-)t}$$

$$30 \quad \rho_{11}(0) = 0 \Rightarrow \rho_{11}(t) = \rho_{11}^{st} (1 - e^{-(\Gamma_+ + \Gamma_-)t})$$



31 Irreversible decay: the Lindblad form describes the coupling to a continuum.