Grover’s Algorithm, Quantum Fourier Transform, and Resource Estimates

Master QLMN - Experimental quantum computing

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Quiz: NMR Implementation of Grover's Algorithm

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Q1: Physical Hamiltonian and Qubit Encoding

A

The experiment was conducted using a homonuclear spin system at cryogenic temperatures.

B

The two qubits were represented by nuclear spins of ^{1}H and ^{13}C in chloroform molecules.

C

The interaction term in the Hamiltonian is of the Ising form $H_{int} \propto I_{zA} I_{zB}$.

D

The scalar coupling constant J was approximately 215 Hz.

Q1: Solution

A

The experiment was conducted using a homonuclear spin system at cryogenic temperatures.

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The two qubits were represented by nuclear spins of ^{1}H and ^{13}C in chloroform molecules.

C

The interaction term in the Hamiltonian is of the Ising form $H_{int} \propto I_{zA} I_{zB}$.

D

The scalar coupling constant J was approximately 215 Hz.

Correct Answers: B, C, D

- The experiment used a **heteronuclear** system (^{1}H and ^{13}C) in chloroform at **room temperature** (not cryogenic).
- The Hamiltonian (Eq. 1 in the paper) is dominated by the scalar coupling term $2\pi\hbar J I_{zA} I_{zB}$.
- The measured coupling J was 215 Hz.

Q2: Pure State Preparation

A

The sample was cooled to the ground state using optical pumping.

B

A technique called “temporal labeling” was used, which involves summing results of three separate experiments with permuted populations.

C

The experiment utilized a single-shot projective measurement to collapse the ensemble into the ground state.

D

The effective pure state was defined using deviation density matrices.

Q2: Solution

A

The sample was cooled to the ground state using optical pumping.

B

A technique called “temporal labeling” was used, which involves summing results of three separate experiments with permuted populations.

C

The experiment utilized a single-shot projective measurement to collapse the ensemble into the ground state.

D

The effective pure state was defined using deviation density matrices.

Correct Answers: B, D

- True pure state preparation is difficult in NMR at room temperature.
- The authors used **“temporal labeling”** (Eq. 2 in the paper) to simulate a pure state by permuting populations across three experiments.
- NMR measures the **deviation density matrix** (traceless part), effectively ignoring the identity component.

Q3: Grover Operator and Oracle Implementation

A

The conditional phase shift (Oracle) was implemented using the natural scalar coupling evolution.

B

For $N = 4$, the algorithm requires $\frac{\pi}{4}\sqrt{N} \approx 1.57$ iterations.

C

The “inversion about the mean” operator D was compiled as $D = WPW$.

D

The Walsh-Hadamard transform was applied using $H = X\bar{Y}_{\frac{1}{2}}$.

Q3: Solution

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The conditional phase shift (Oracle) was implemented using the natural scalar coupling evolution.

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The “inversion about the mean” operator D was compiled as $D = WPW$.

D

The Walsh-Hadamard transform was applied using $H = X\bar{Y}_{\frac{1}{2}}$.

Correct Answers: A, C, D

- The oracle uses the coupled-spin evolution (free evolution).
- For $N = 4$, the exact solution is found in **one** iteration (not 1.57).
- The inversion operator D is decomposed into Walsh-Hadamard transforms sandwiching a conditional phase shift (Eq. 5 in the paper).

Q4: Algorithm Dynamics and Periodicity

A

The amplitude of $|11\rangle$ reaches a maximum after exactly 1 iteration.

B

The density matrix exhibits a periodicity of 3 iterations.

C

The algorithm is unstable and loses periodicity after the first cycle due to decoherence.

D

The classical expected number of queries is 2.25, whereas quantum solved it in a single step.

Q4: Solution

A

The amplitude of $|11\rangle$ reaches a maximum after exactly 1 iteration.

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The density matrix exhibits a periodicity of 3 iterations.

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The algorithm is unstable and loses periodicity after the first cycle due to decoherence.

D

The classical expected number of queries is 2.25, whereas quantum solved it in a single step.

Correct Answers: A, B, D

- The paper confirms that for $N = 4$, the answer is found in 1 step (Eq. 6).
- The periodicity of the probability is 3 iterations (Fig. 1 in the paper).
- The classical average for searching 4 items is indeed $(1 + 2 + 3 + 4)/4 \approx 2.25$.

Q5: Read-out Method and Error Analysis

A

Full state tomography was used, requiring 9 different rotation experiments.

B

The longest computation (7 iterations) was significantly longer than T_2.

C

Signal strength represents the fraction of population in a given state.

D

The primary error source (7-44%) was magnetic field inhomogeneity.

Q5: Solution

A

Full state tomography was used, requiring 9 different rotation experiments.

B

The longest computation (7 iterations) was significantly longer than T_2.

C

Signal strength represents the fraction of population in a given state.

D

The primary error source (7-44%) was magnetic field inhomogeneity.

Correct Answers: A, C, D

- Readout in ensemble NMR measures magnetization (expectation values), not collapse.
- Tomography required 9 settings (27 total runs with temporal labeling).
- The computation time (35ms) was well **within** $T_2 \approx 300 - 400$ ms, not longer.
- Field inhomogeneity was cited as the primary error source.

Grover's Algorithm

Algorithm Overview

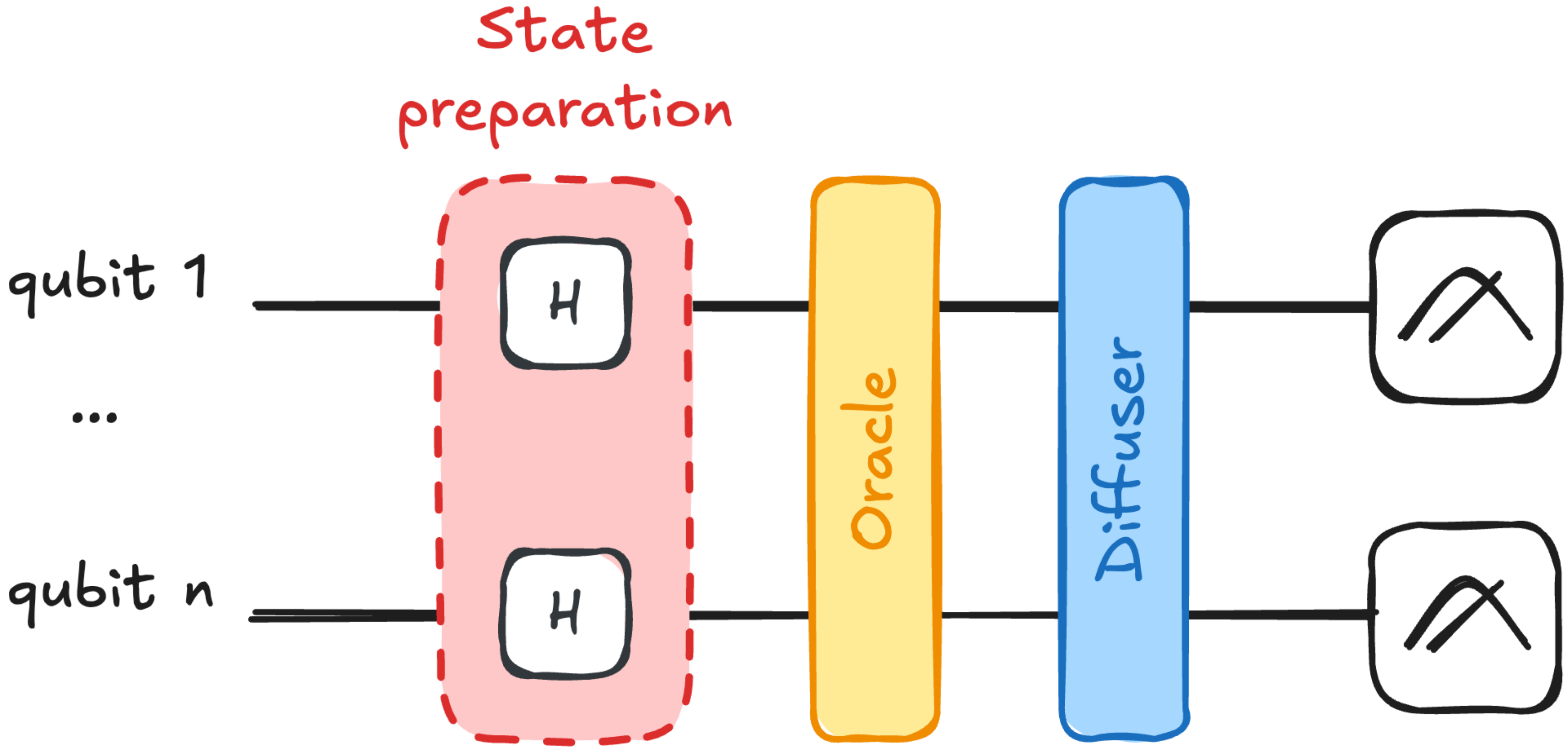

Figure 1: Grover's algorithm circuit showing initialization, oracle, and diffusion operator

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Amplitude Evolution

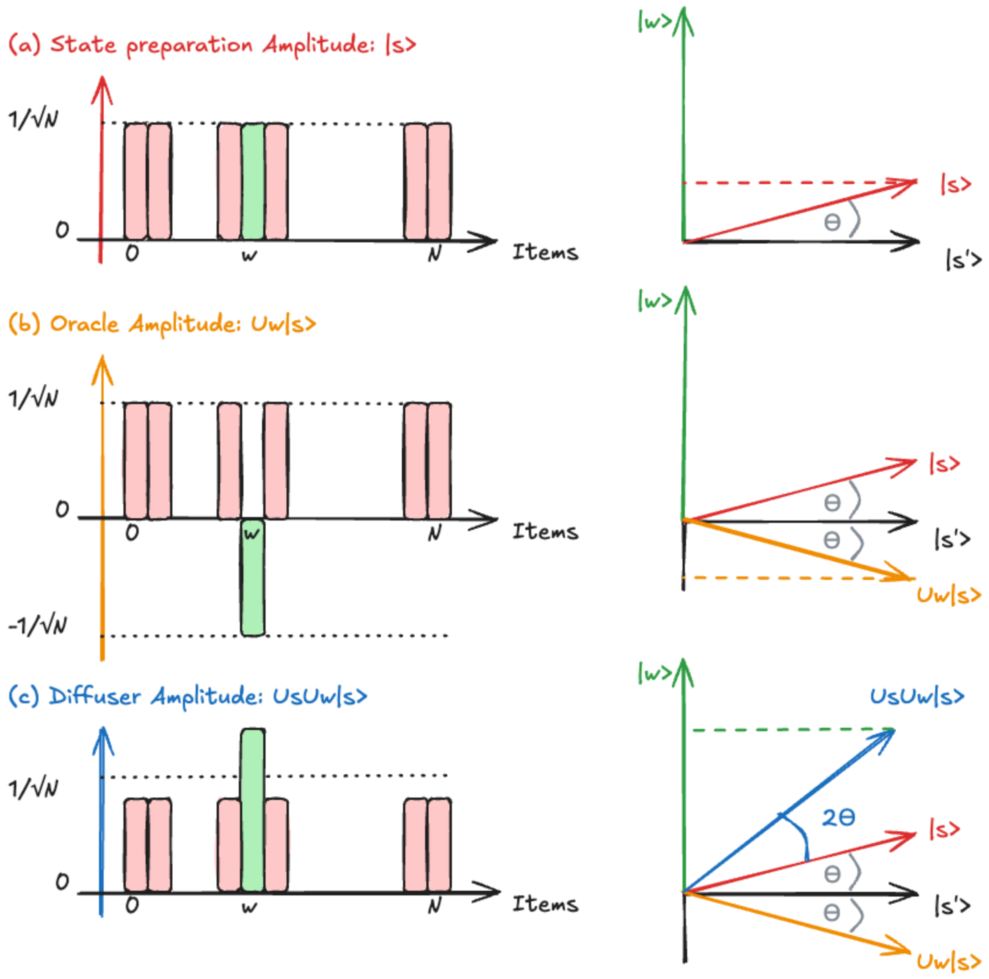

Figure 2: Amplitude evolution during Grover's algorithm showing state preparation (red), oracle (orange) and diffusion (blue) steps

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Geometric View: Two Reflections

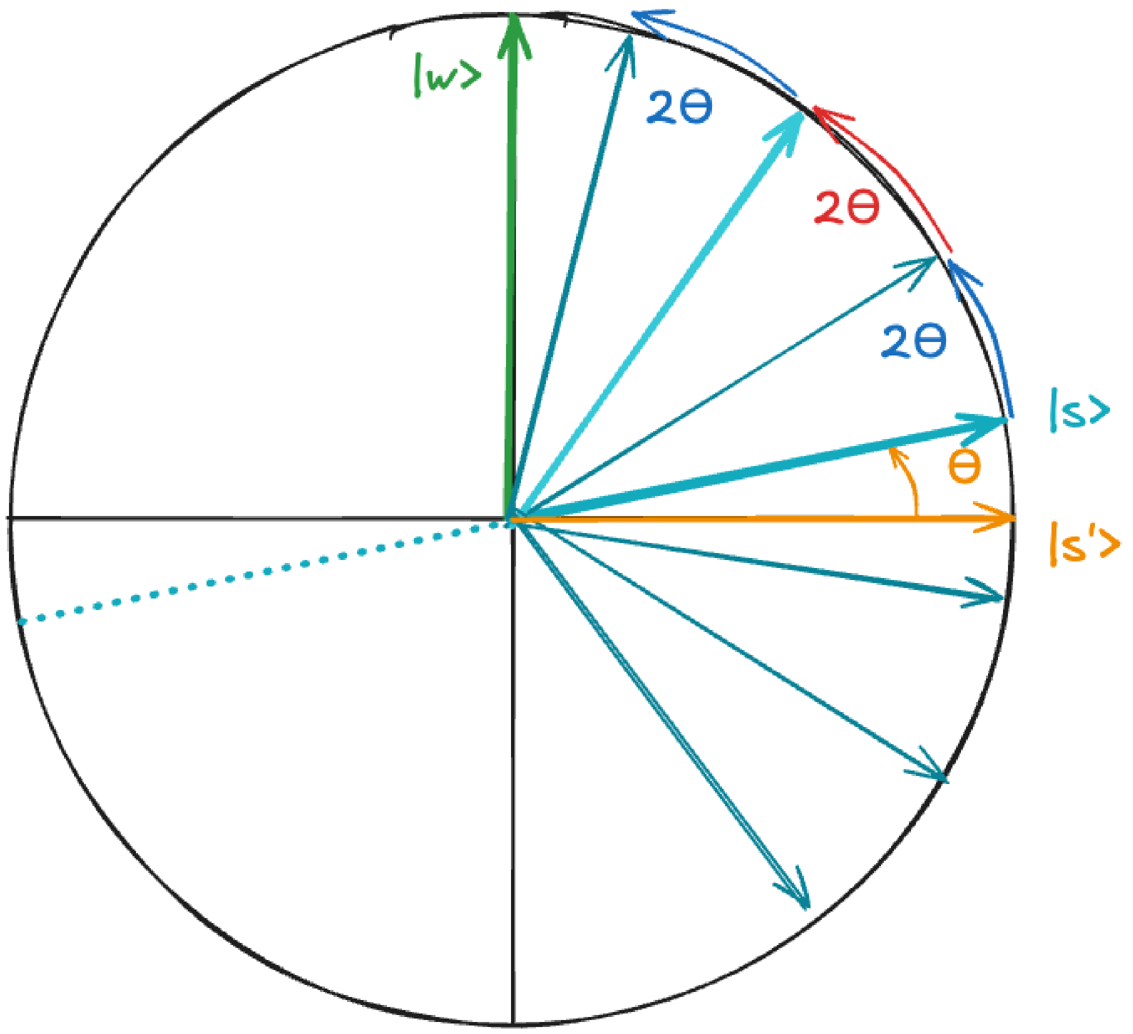

Figure 3: Geometric view of Grover's algorithm as rotations in 2D subspace

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2-Qubit Example: Complete Circuit

For $N = 4$ with winner $|11\rangle$: single iteration gives perfect success!

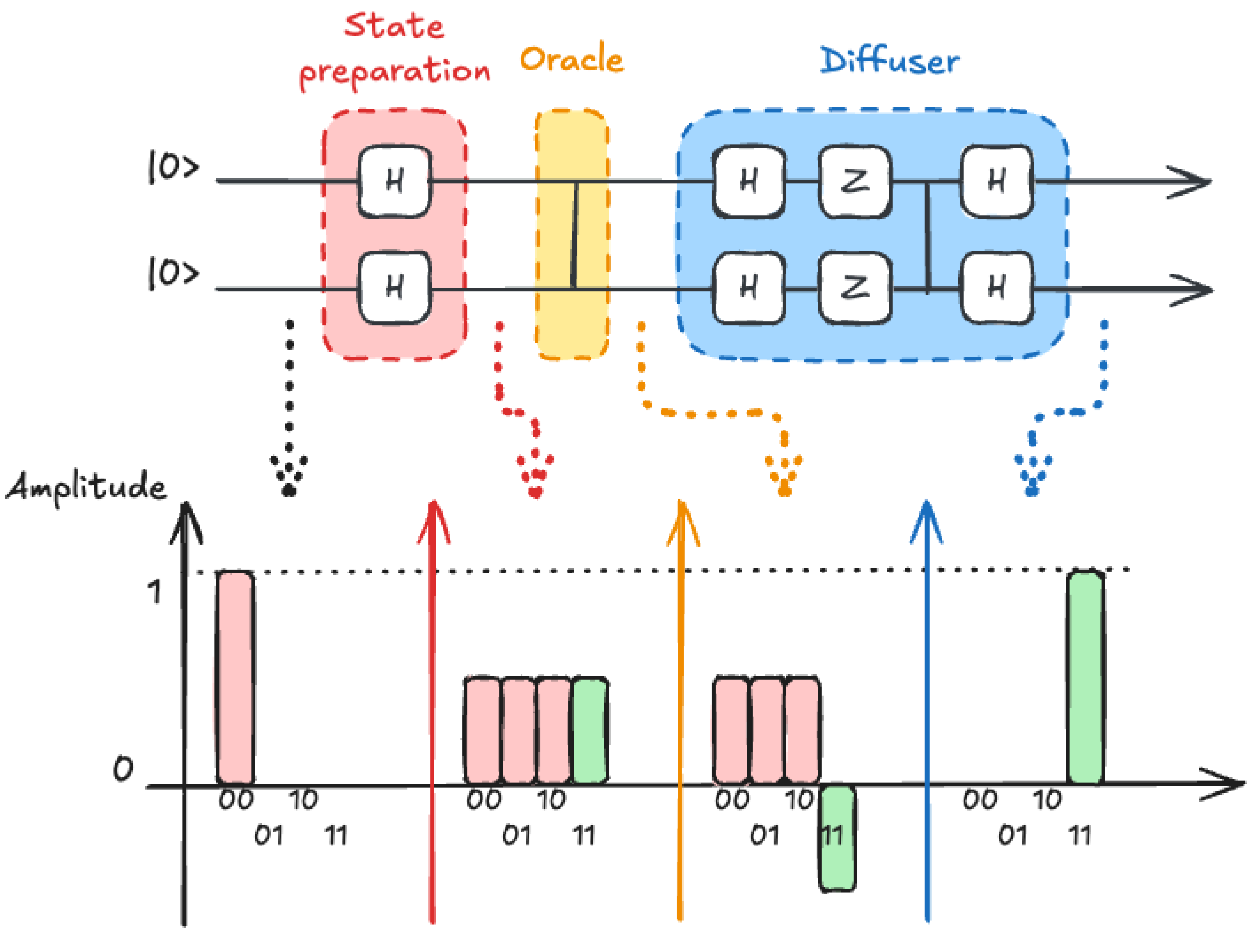

Figure 4: Detailed circuit implementation of Grover's algorithm with gate decomposition

Qiskit Examples

2 and 3 Qubits

Go to **Colab** and open: **grover.ipynb**

Application: Sudoku

How would you encode a 2×2 Sudoku as a Grover search?

Let's try it in Qiskit!

Experimental Implementations

Platform	Qubits	Exp. Success	Theo. Success	Reference
SC	3	-	-	AbuGhanem (2025)
SC	6	4%	-	Chu et al. (2023)
SC	4	45%	-	Chu et al. (2023)
Trapped Ions	3	39%	78%	Figgatt et al. (2017)
Photonics	2	71%	-	Main et al. (2025)
Silicon Spins	3	93%	98%	Thorvaldson et al. (2025)
NMR	2	-	-	Chuang, Gershenfeld, and Kubinec (1998)

Quantum Fourier Transform

Classical Fourier Transform

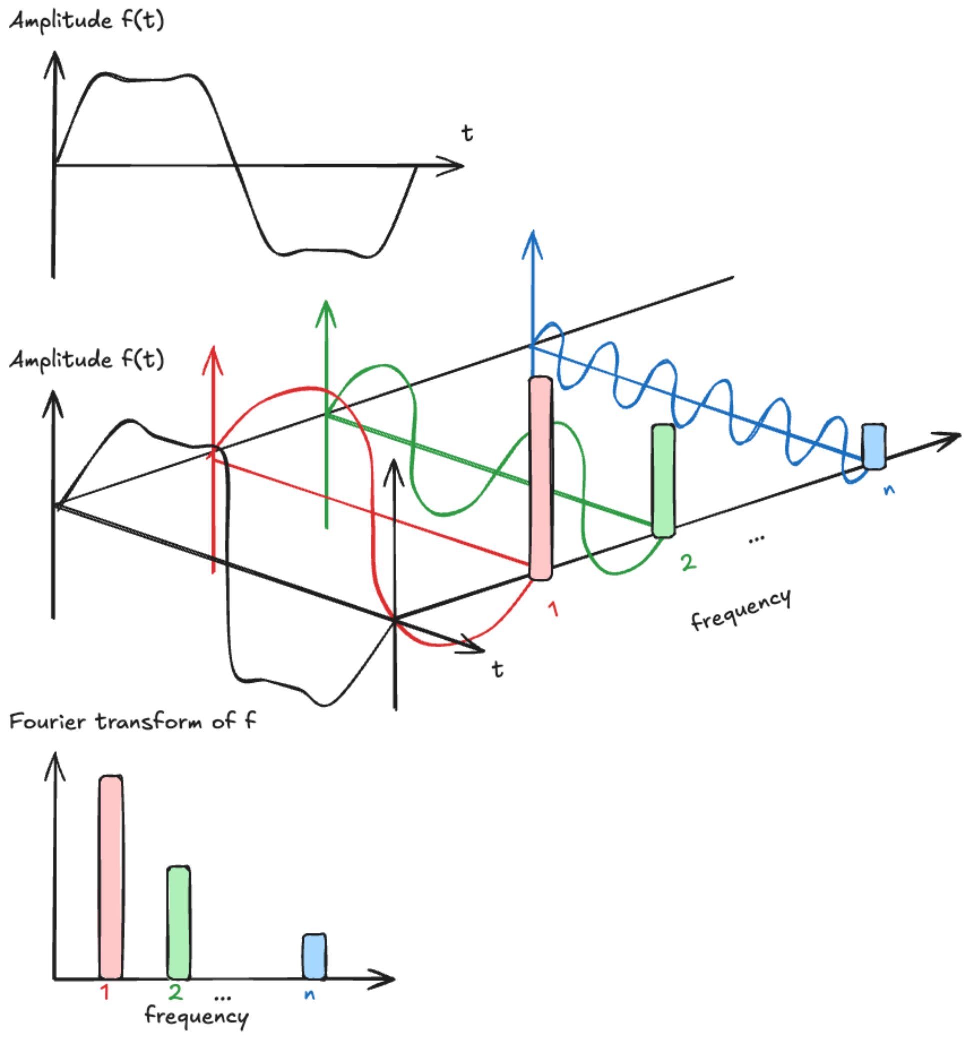

Figure 5: Fourier transform: time domain (black) decomposed into frequency components (red, green, blue)

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Visualizing QFT:

Z-Basis vs Fourier Basis Counting

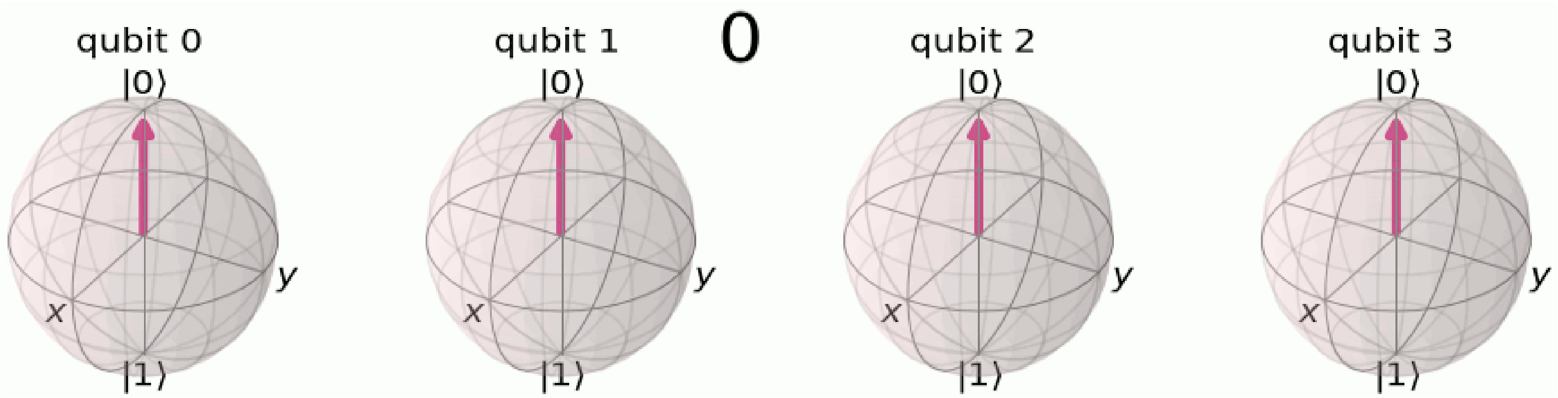

Figure 6: Representation of basis states on 4 qubits

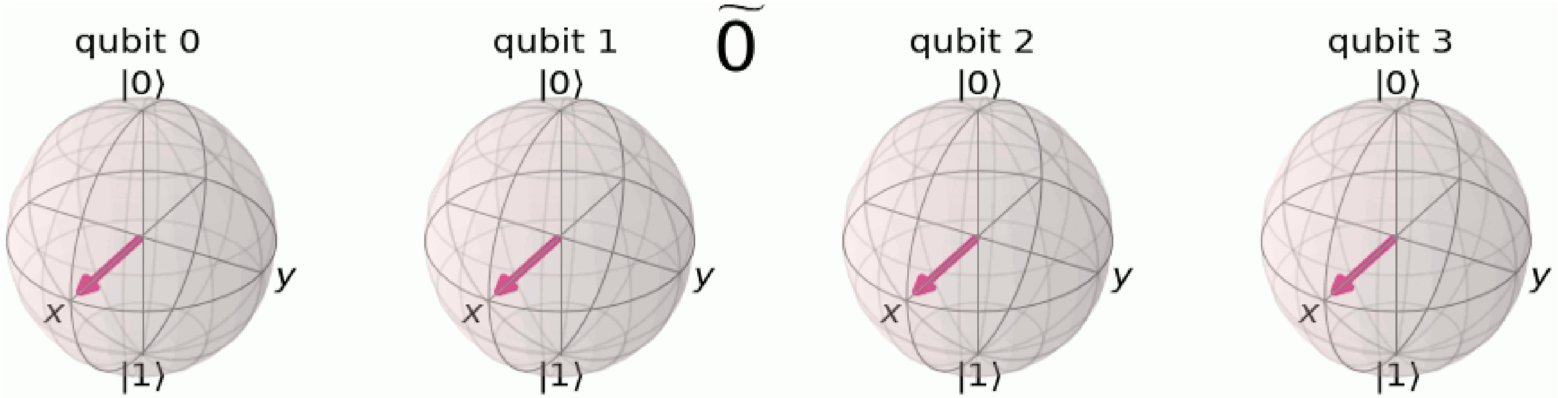

Figure 7: Representation of Fourier basis states on 4 qubits

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QFT Circuit Implementation

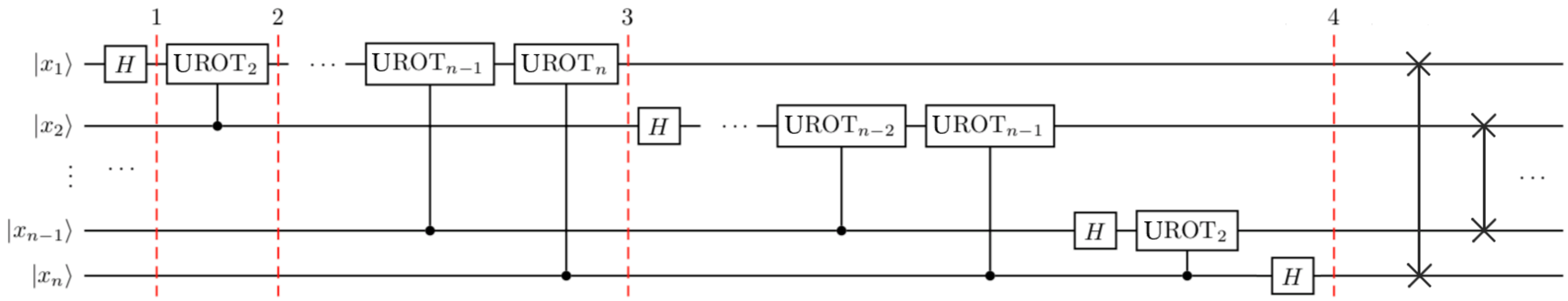

Figure 8: QFT circuit showing Hadamard gates and controlled rotation gates

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Qiskit

Colab + qiskit-demo/presentations/quantum-fourier-transform.ipynb at main · francois-marie/qiskit-demo

Quantum Phase Estimation

QPE Circuit

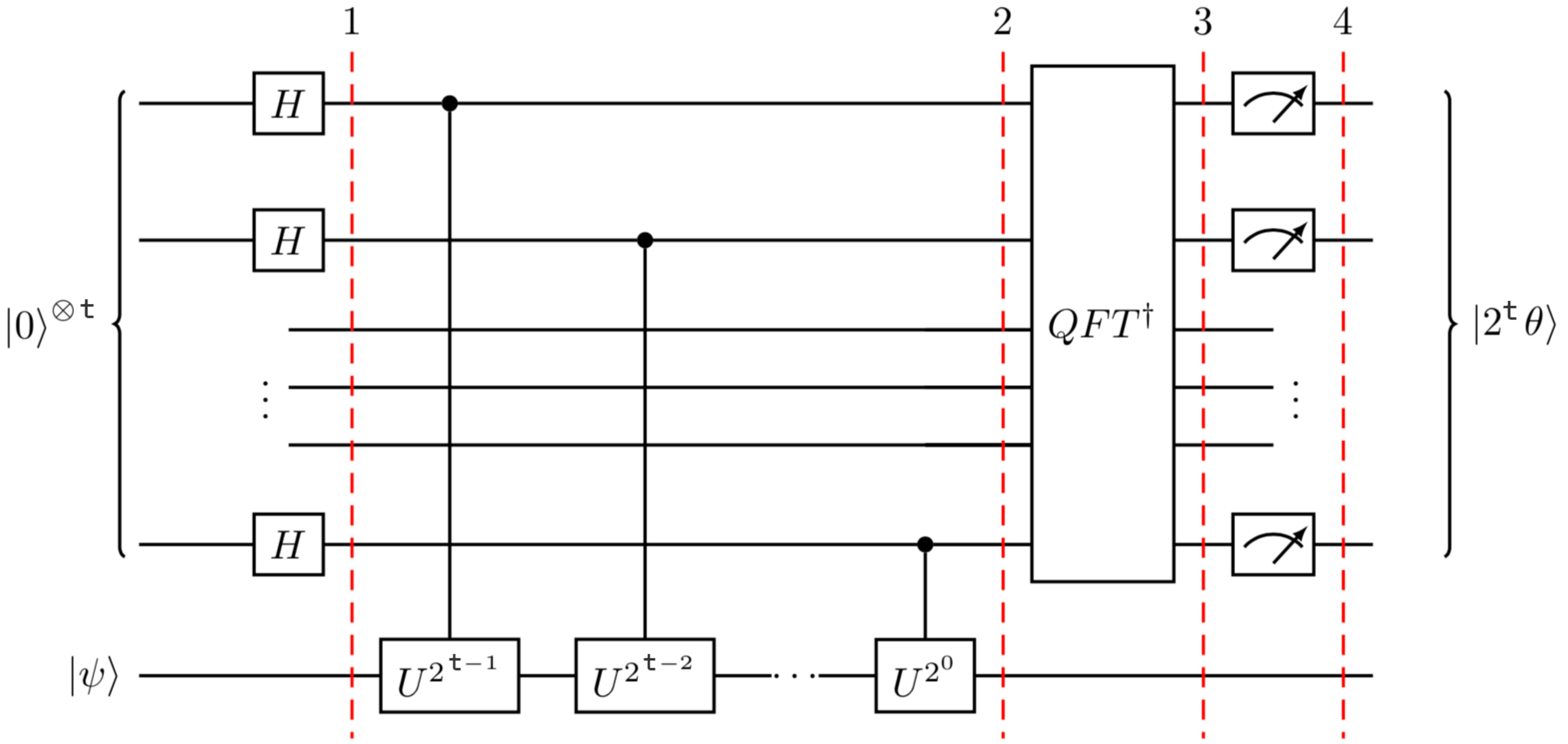

Figure 9: Quantum Phase Estimation circuit with controlled-U gates and inverse QFT

Qiskit

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Resource Estimates

Quantum Algorithm Landscape

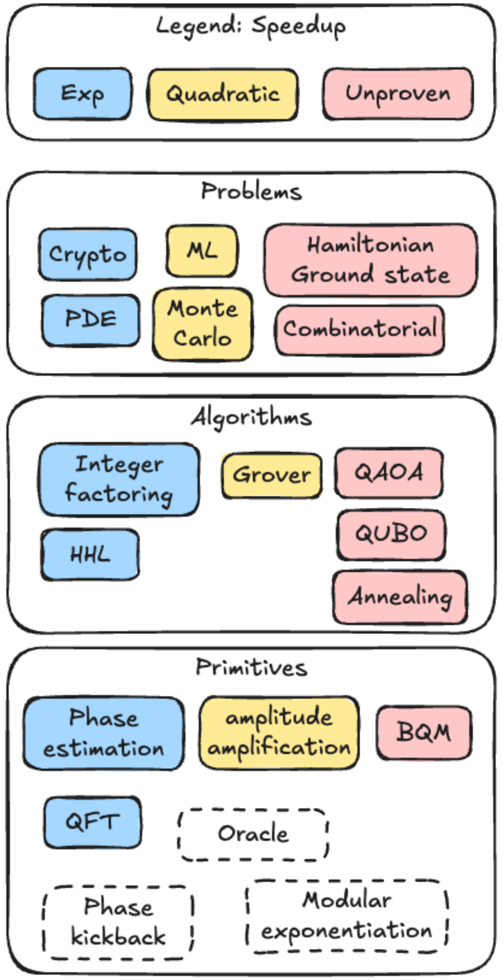

Figure 10: Partial landscape of quantum algorithms showing primitives, algorithms, problem classes and speedup types

Resource Requirements

Application	Logical Qubits	Logical Gates	Physical Qubits	Runtime
FeMoCo (Chemistry)	100-1,000	10^{14}	-	Days-months
Financial optimization	10^4	10^{10}	-	-
RSA-2048 (SC)	$\sim$1,000	10^{10}	$\sim$1 Million	$\sim$1 week
RSA-2048 (NA)	$\sim$10,000	10^{10}	$\sim$20 Million	$\sim$1 week

Historical Progress

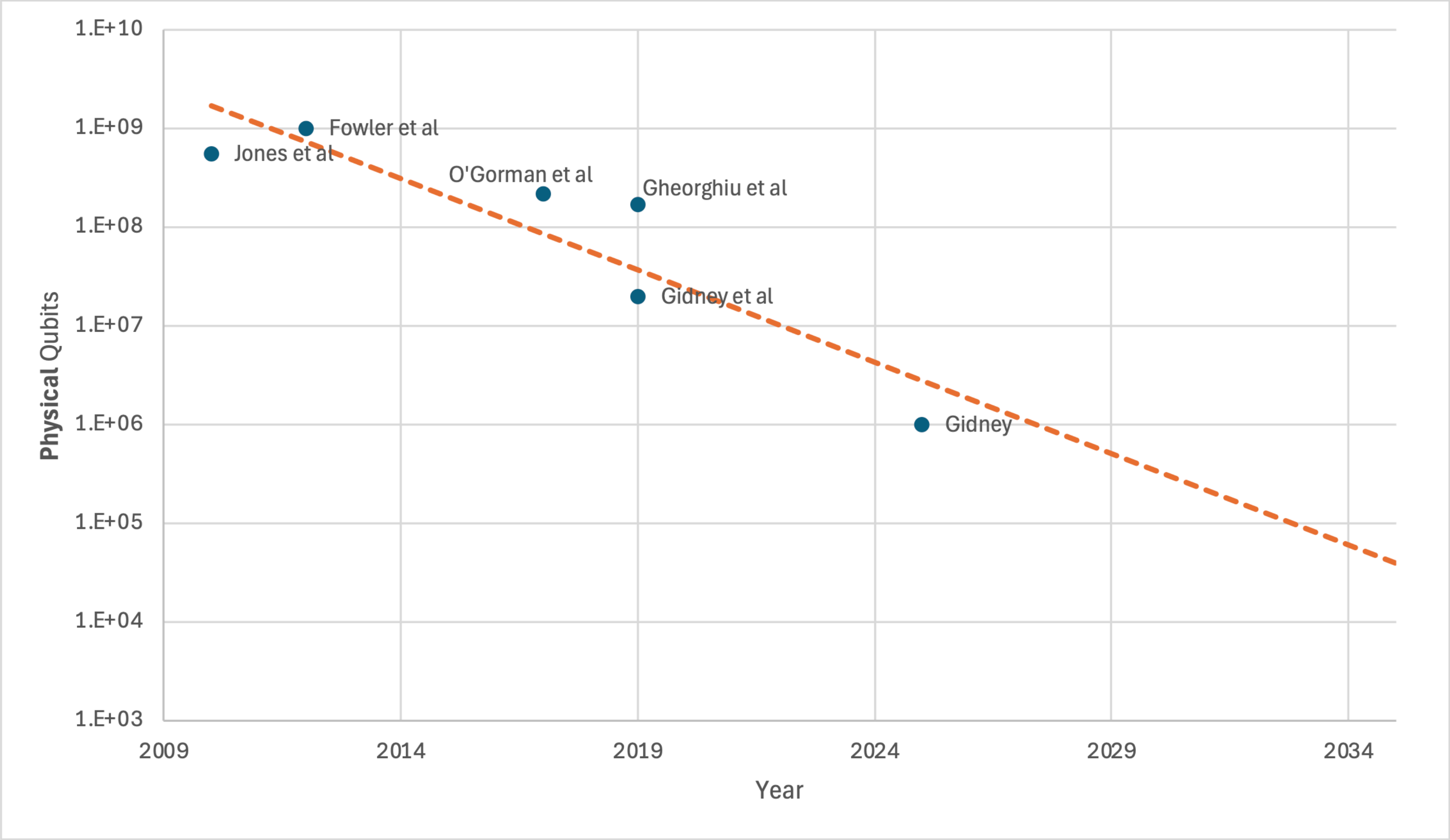

Figure 11: Historical progress in reducing resource estimates for cryptographically-relevant quantum algorithms

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Where We Stand Today

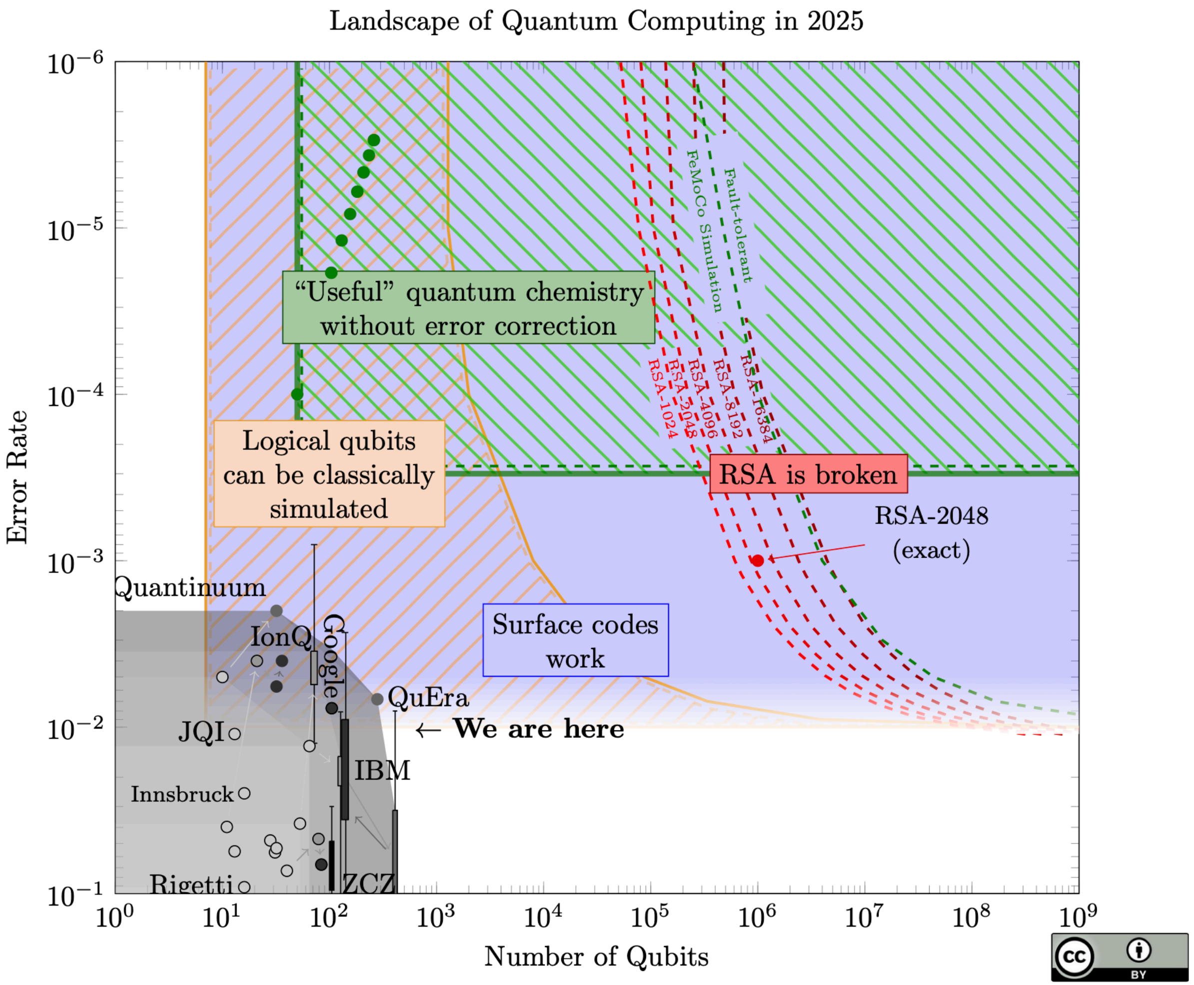

Figure 12: Current quantum computing landscape showing the gap between available hardware and algorithm requirements

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Thank you for your attention!

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