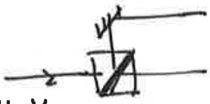
Homework 3 - photonic qbits

Exo 1 Consider  } dual rail encoding.

A polarization BS transmits a V polarization and reflects a H polarization.

H $\longrightarrow$  $|0\rangle$ $|1\rangle$

V $\longrightarrow$  $|0\rangle$ $|1\rangle$

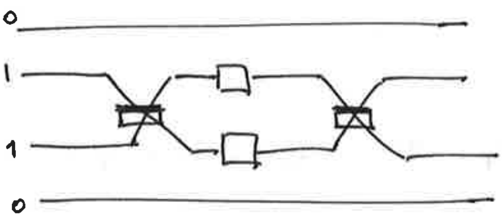
Exo 2 Action of the $-\pi/2$ waveplate: $U_{wp} = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}_{\substack{a,b \\ d,c}}$

Hence the optical setup is described by the unitary operator:

$$U = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}_{c,d}}_{wp\,2} \underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}}_{BS} \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}_{ab}}_{wp\,1} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}$$

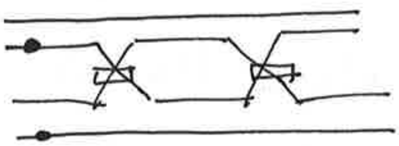
$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = U_H.$$

Exo 4 There was a mistake in the drawing of fig 3b: it should have been:



Besides, the convention for the BS should have been $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

for: • 00 $\longrightarrow$ 00 corresponding to 

• 10 

for the photon in b.

$|1_b 0_c\rangle \xrightarrow{BS1} \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$

$\xrightarrow{BS2} \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$

$+ \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}(|10\rangle - |01\rangle)$

$= |1_{b'} 0_{c'}\rangle$

The shiftings are irrelevant for 1 photon in the interferometer.

Hence $|1\,0\rangle \longrightarrow |10\rangle$

• for $|01\rangle = |n_a=1, n_b=0\rangle|n_c=1, n_d=0\rangle$, same argument

$\rightarrow |01\rangle$

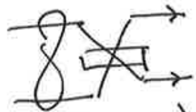
• for $|11\rangle = |n_a=0, n_b=1\rangle|n_c=1\ n_d=0\rangle$

look only at the photon in the interferometer.

$$|11\rangle_{bc} \xrightarrow{BS_1} \frac{1}{\sqrt2}(|20\rangle - |02\rangle) \xrightarrow{NS} -\frac{1}{\sqrt2}(|20\rangle - |02\rangle)$$

$$\xrightarrow{BS_2} -\frac{1}{\sqrt2}\left[\frac{1}{2}|20\rangle + \frac12|02\rangle + \frac{1}{\sqrt2}|11\rangle\right]$$
$$+\frac{1}{\sqrt2}\left[\frac12|20\rangle+\frac12|02\rangle - \frac1{\sqrt2}|11\rangle\right] = -|11\rangle$$

(you have to use the BS in reverse

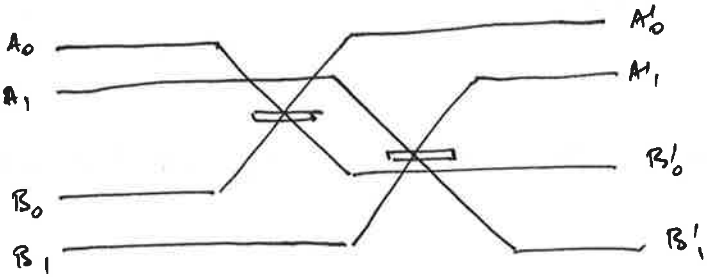


$\frac1{\sqrt2}(|20\rangle-|02\rangle)$

To summarize $|Q_1Q_2\rangle \to (-1)^{Q_1\cdot Q_2}|Q_1Q_2\rangle$

which is a CZ gate.

Exo 5



One calculates first the action of the BS on each of the 2 qbit states.

• $|00\rangle = |1_{A_0}0_{A_1}\rangle|1_{B_0}0_{B_1}\rangle = |1_{A_0}1_{B_0}\rangle|0_{A_1}0_{B_1}\rangle$

$\to \frac1{\sqrt2}(|2_{A'_0}0_{B'_0}\rangle - |0_{A'_0}2_{B'_0}\rangle)|0_{A'_1}0_{B'_1}\rangle = \frac1{\sqrt2}(|2_{A'_0}0_{A'_1}0_{B'_0}0_{B'_1}\rangle$
$- |0_{A'_0}0_{A'_1}2_{B'_0}0_{B'_1}\rangle$

• $|11\rangle = |0_{A_0}1_{B_0}\rangle|1_{B_0}1_{B_1}\rangle$

$\to \frac1{\sqrt2}(|0_{A'_0}2_{A'_1}0_{B'_0}0_{B'_1}\rangle - |0_{A'_0}0_{A'_1}0_{B'_0}2_{B'_1}\rangle)$

• $|01\rangle = |1_{A_0}0_{A_1}\rangle|0_{B_0}1_{B_1}\rangle = |1_{A_0}0_{B_0}\rangle|0_{A_1}1_{B_1}\rangle$

$\to \frac1{\sqrt2}(|1_{A'_0}0_{B'_0}\rangle + |0_{A'_0}1_{B'_0}\rangle)\frac1{\sqrt2}(|1_{A'_1}0_{B'_1}\rangle - |0_{A'_1}1_{B'_1}\rangle)$

$= \frac12(|1_{A'_0}1_{A'_1}0_{B'_0}0_{B'_1}\rangle - |1_{A'_0}0_{A'_1}0_{B'_0}1_{B'_1}\rangle + |0_{A'_0}1_{A'_1}1_{B'_0}0_{B'_1}\rangle - |0_{A'_0}0_{A'_1}1_{B'_0}1_{B'_1}\rangle)$

• $|10\rangle \to \frac12(|1_{A'_0}1_{A'_1}0_{B'_0}0_{B'_1}\rangle + |1_{A'_0}0_{A'_1}0_{B'_0}1_{B'_1}\rangle - |0_{A'_0}1_{A'_1}1_{B'_0}0_{B'_1}\rangle - |0_{A'_0}0_{A'_1}1_{B'_0}1_{B'_1}\rangle)$

Hence $|\phi_\pm\rangle \propto |00\rangle \pm |11\rangle$ gives at the output 4 possible patterns, identical for ϕ_+ and ϕ_- :

2 0 0 0
0 2 0 0
0 0 2 0
0 0 0 2

On the Contrary: $|\psi_\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$ give 2 different patterns at the output:

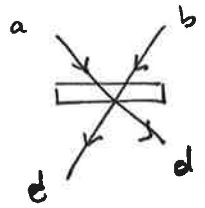
$\psi_+ \rightarrow$ 1 1 0 0 or 1 0 0 1

$\psi_- \rightarrow$ 1 0 0 1 or 0 1 1 0

They can now be distinguished.

Proba of success: $2/4 = 1/2$

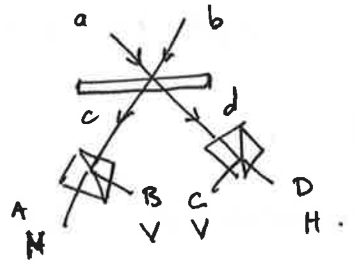
Exo 6 Polarization encoding and Bell states analysis



If $|\psi_{in}\rangle = |\phi_\pm\rangle = \frac{1}{\sqrt{2}}(|HH\rangle \pm |VV\rangle)$

then $|\psi_{out}\rangle = \frac{1}{2}(|2_H 0_H\rangle_{cd} - |0_H 2_H\rangle_{cd})$
$\pm \frac{1}{2}(|2_V 0_V\rangle_{cd} - |0_V 2_V\rangle_{cd})$.

The PBS allows polarization detection.



Hence the click patterns are the same for $\phi_\pm$

	A	B	C	D
$2_H 0_H$	x	0	0	0
$0_H 2_H$	0	0	0	x
$2_V 0_V$	0	x	0	0
$0_V 2_V$	0	0	x	0

When instead $|\psi_{in}\rangle = |\psi_\pm\rangle = \frac{1}{\sqrt{2}}(|HV\rangle \pm |VH\rangle)$, the bunching effect does not take place and after the BS :

$$|\psi_{out}\rangle \propto (H_c + H_d)(V_c - V_d) \pm (V_c + V_d)(H_c - H_d)$$

$\Rightarrow$ ψ_+ gives $\frac{1}{\sqrt{2}}(H_c V_c - H_d V_d)$ clic : AB or CD

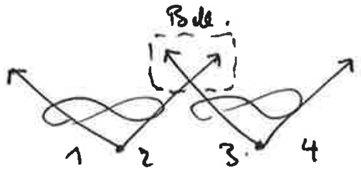
ψ_- gives $\frac{1}{\sqrt{2}}(- H_c V_d + H_d V_c)$ clic : AC or BD.

Hence can be distinguished.

Again Proba of success: 2 out of 4 times (distinguish 2 states out of 4)

Exo 7 Entanglement swapping.

Recall
$$|00\rangle = \frac{1}{\sqrt{2}}\left(|\psi_+\rangle + |\psi_-\rangle\right) \qquad |11\rangle = \frac{1}{\sqrt{2}}\left(|\psi_+\rangle - |\psi_-\rangle\right)$$
$$|01\rangle = \frac{1}{\sqrt{2}}\left(|\phi_+\rangle + |\phi_-\rangle\right) \qquad |10\rangle = \frac{1}{\sqrt{2}}\left(|\phi_+\rangle - |\phi_-\rangle\right)$$



$$|\psi\rangle_{12}\,|\psi\rangle_{34}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle_{12} + |11\rangle_{12}\right)\frac{1}{\sqrt{2}}\left(|00\rangle_{34} + |11\rangle_{34}\right)$$

$$|\psi_{12}\rangle|\psi_{34}\rangle = \frac{1}{2}\left(|00\rangle_{14}|00\rangle_{23} + |01\rangle_{14}|01\rangle_{23} + |10\rangle_{14}|10\rangle_{23} + |11\rangle_{14}|11\rangle_{23}\right)$$

$$\propto \frac{1}{2}\Big[\,|00\rangle_{14}\left(|\psi_+\rangle_{23} + |\psi_-\rangle_{23}\right) + |01\rangle_{14}\left(|\phi_+\rangle_{23} + |\phi_-\rangle_{23}\right)$$
$$+\ |10\rangle_{14}\left(|\phi_+\rangle_{23} - |\phi_-\rangle_{23}\right) + |11\rangle_{14}\left(|\psi_+\rangle_{23} - |\psi_-\rangle_{23}\right)\Big]$$

$$\propto \left(|00\rangle_{14} + |11\rangle_{14}\right)|\psi_+\rangle_{23} + \left(|00\rangle_{14} - |11\rangle_{14}\right)|\psi_-\rangle_{23}$$
$$+ \left(|01\rangle_{14} + |10\rangle_{14}\right)|\phi_+\rangle_{23} + \left(|01\rangle_{14} - |10\rangle_{14}\right)|\phi_-\rangle_{23}$$

$$\propto |\psi_+\rangle_{14}\,|\psi_+\rangle_{23} + |\psi_-'\rangle_{14}\,|\psi_-\rangle_{23} + |\phi_+\rangle_{14}\,|\phi_+\rangle_{23} + |\phi_-\rangle_{14}|\phi_-\rangle_{23}$$

A Bell measurt giving for example $|\psi_+\rangle_{23}$ projects the qbit 14 into the entangled state $|\psi_+\rangle$, with a probality $1/4$.

As the Bell meast has a success probability $1/2$, the probability of the entanglement swapping is $1/4 \times 1/2 = 1/8$.

Exo 8

$$F_p = \frac{1{,}3 \text{ ns}}{0.16 \text{ ns}} \approx 8$$

As $F_p = \frac{3}{4\pi^2}\left(\lambda/n\right)^3 \frac{Q}{V}$ $\Rightarrow$ $V = \frac{3}{4\pi^2}\left(\frac{890}{3{,}5}\right)^3 \frac{12000}{8}$

$$= \frac{3}{4\pi^2}\frac{1}{3{,}5^3}\frac{12000}{8}\lambda^3 \approx 3\lambda^3$$

Exo 9

$$g^{(2)}(0) = \frac{\langle a^+a^+aa\rangle}{\langle a^+a\rangle^2} = \frac{\langle a^+(1-aa^+)a\rangle}{\langle a^+a\rangle^2} \quad \text{using } [a,a^+]=1$$

Hence: $g^{(2)}(0) = \frac{\langle \hat{n}(\hat{n}-1)\rangle}{\langle \hat{n}\rangle^2} = \frac{\sum_n n(n-1)P_n}{\left(\sum_n nP_n\right)^2}$

When $P_1, P_2 \ldots \ll 1$ $\quad g^{(2)}(0) \approx \frac{2(2-1)P_2}{(P_1)^2} = \frac{2P_2}{P_1^2}$

For a coherent state $|\alpha\rangle = e^{-|\alpha|^2/2}\sum \frac{\alpha^n}{\sqrt{n!}}|n\rangle$

and $P_1 = \alpha^2 e^{-\alpha^2} \simeq \alpha^2$ $\quad P_2 = \frac{\alpha^4}{2}e^{-\alpha^2} \simeq \frac{\alpha^4}{2}$

Hence: $\frac{2P_2}{P_1^2} \simeq \frac{\alpha^4}{(\alpha^2)^2} = 1$

Exo 10

take $|\psi\rangle = |0_a 0_b\rangle + \varepsilon|1_a 1_b\rangle + \varepsilon^2|2_a 2_b\rangle$

A conditional measurement of a photon on b projects onto

$$\hat{b}|\psi\rangle \propto |1_a\rangle + \varepsilon\sqrt{2}\,|2_a\rangle$$

$\Rightarrow$ $g^{(2)}(0) = \frac{\langle \hat{n}(\hat{n}-1)\rangle}{\langle \hat{n}\rangle^2} \approx \frac{2P_2}{P_1^2} \simeq 4\varepsilon^2$