Master QLMN Antoine Browaeys
Experimental quantum computing antoine.browaeys@institutoptique.fr

Lecture 3 - Spin qubits

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Probably the cleanest two-level system, apart from the polarization of photons, is a spin 1/2. The qubit encoding is then easy: $|0\rangle = |\downarrow\rangle$ and $|1\rangle = |\uparrow\rangle$. It can be the spin of an electron or the one of a nucleus for which $I = 1/2$ such as ^{1}H, ^{13}C or ^{15}N for example. As any spin $\mathbf{J}$ is associated to a magnetic moment $\boldsymbol{\mu} = \gamma\mathbf{J}$, it can be manipulated using oscillating magnetic fields. For example, placing an electron spin in a $B = 1\,\mathrm{T}$ field yields a frequency between spin up and down of 2.8 GHz, and microwave at this frequency can induce transitions from up to down. On the contrary, a proton in the same field gives a transition frequency of 21 MHz, and can thus be manipulated by radio-frequency (RF).

We will describe very briefly here two approaches to quantum computing using either nuclear spins or electron spins.

1 Liquid nuclear magnetic resonance [1]

This approach was the first one considered for a practical implementation of quantum computing and led to the first demonstration of the factorization of 15 by the Shor's algorithm in 1999. Today, it is no longer the leading approach but its role in the development of experimental quantum computing was crucial, as it inspired all the other approaches. The reason for that comes from the fact that Nuclear Magnetic Resonance techniques have been developed intensively since 1950's: in quantum computing language, they amount to performing single and two-qubit gates, and to controlling decoherence through the dynamical decoupling techniques that we discussed in previous lectures.

Principle of NMR quantum computing. A liquid contains molecules (often custom designed but not always), examples of which are shown in Fig. 1(a). The solution is placed in a high magnetic field, typically 10 T, in a cryostat at a Kelvin temperature (Fig. 1b). The qubits are encoded on the spins of the nuclei, chosen so that $I = 1/2$. The associated nuclear Bohr magneton is $\mu_\mathrm{N} = e\hbar/(2M_\mathrm{N}) = \mu_\mathrm{B}/1836 \approx 7.6\,\mathrm{MHz/T}$, leading to a qubit frequency around 50 MHz depending on the Landé factor of the nucleus considered.

Two choices are possible: either code the qubits on nuclei of different atoms, which will then all have different magnetic moments as they have different Landé factors, or code on nuclei of the same type of atom (e.g. ^{1}H or ^{13}C in an organic molecule). To distinguish the qubits frequencies, we rely on the fact that the chemical environment of each nucleus will be different depending on where it is placed in the molecule (chemical shift). Examples of chemical shifts are given in the Table below, for typical molecules placed in a 10 T magnetic field:

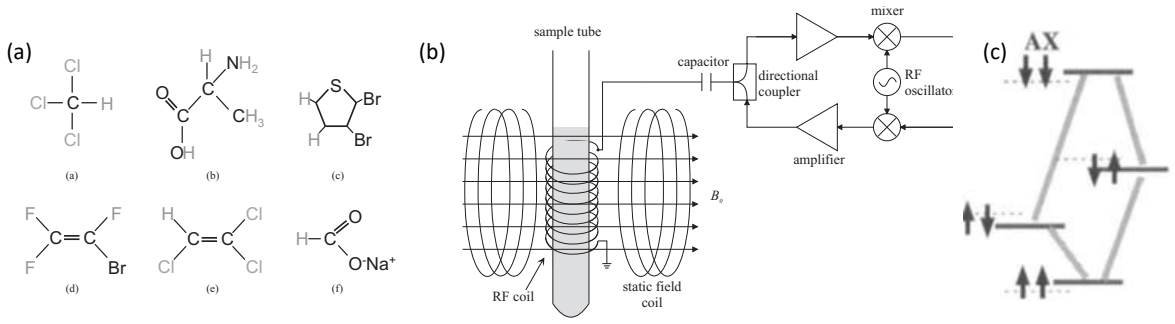

Figure 1: *(a) Examples of molecules used in Liquid NMR quantum computing. The atoms in black are the ones onto which the nuclear qubits are encoded. (b) Schematic of the setup used LNMR with the detection circuit (From Ref. [2]). (c) Energy level of four spin configurations shifted due to their interaction and local environment (From Ref. [1]).*

Nuclei	$\Delta\nu/\nu$	$\Delta\nu$ (kHz)
^{1}H	10 ppm	6
^{13}C, ^{15}N	200 ppm	30

The addressability in the register is ensured by tuning the radio-frequency on resonance with a specific nucleus. Each molecule is thus a quantum register and the liquid is a collection of typically 10^{20} of them.

Single-qubit gates. They are realized by applying a radio-frequency field tuned to the frequency ω_i of a given qubit. A magnetic field $B_x(t) = B_1 \cos(\omega t)$ implements, in the rotating wave approximation, the Hamiltonian $\hat{H} = -\hat{\boldsymbol{\mu}} \cdot \mathbf{B} = -g\mu_{\mathrm{N}} B_1 \hat{\sigma}_x$, with g_{L} the Landé factor.

Readout of the qubit state. It relies on a phenomena called free induction decay. The Bloch sphere picture does help here, see Fig. 2(a). Assume you want to measure the state $|0\rangle = |\downarrow\rangle$. Apply a $\pi/2$-pulse around the x axis to prepare a superposition $|0\rangle + |1\rangle$. Following this pulse, the associated Bloch vector, proportional to the magnetic moment of the qubit, is now in the equatorial plan and rotates around the z axis at the free frequency ω_i, and, due to dissipation, decays. A pickup coil, placed perpendicularly to the y-axis, will measure a macroscopic signal corresponding to the rotation of the nuclear spins in all the molecules of the liquid, of the form $S(t) \propto N\langle\hat{\sigma}_y(t)\rangle \propto e^{-\gamma t}\cos(\omega_i t)$. If you want to measure the state $|1\rangle = |\uparrow\rangle$, the same procedure will return $-S(t)$, thus allowing one to distinguish between the two qubit states. Examples of pulses obtained to measure the state of two qubits are shown in Fig. 2(b).

Two-qubit gates. In the molecules, spins from nearby nuclei can interact. There are two ways for them to do so. The direct magnetic dipole interaction $\propto \mu_{\mathrm{N}}^2/R^3$, but usually it averages out to zero owing to the random orientations of the spins with respect to each other. The second one involves an interaction mediated by the electrons of the chemical bound between two atoms A and B, and it ressembles an Ising like interaction

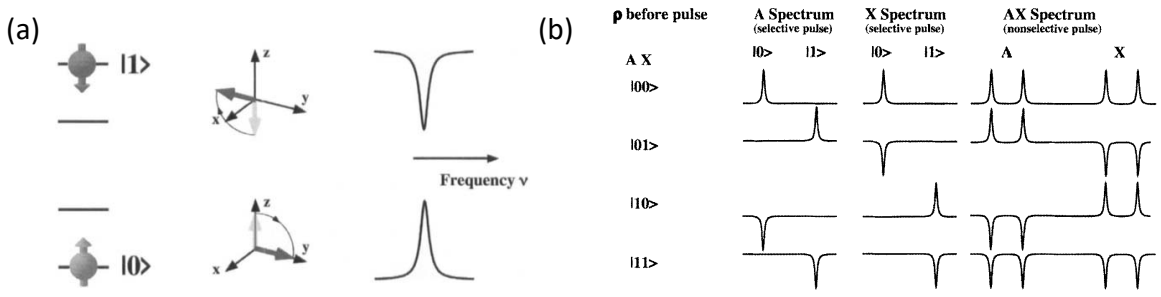

Figure 2: *(a) Principle of the detection of the qubit state by free induction decay. (b) Schematic of the sequence of pulses obtained for the detection of four different 2-qubit states by free induction decay (From Ref. [1]).*

of the form:

$$\hat{H} = A\hat{I}_z^A\hat{I}_z^B \ . \tag{1}$$

This diagonal interaction combined to different local chemical environments leads to different shifts of each of the four two-spin states $|\uparrow,\uparrow\rangle = |1,1\rangle$, $|\uparrow,\downarrow\rangle = |1,0\rangle$, $|\downarrow,\uparrow\rangle = |0,1\rangle$ and $|\downarrow,\downarrow\rangle$, as represented in Fig. 1(c) . Applying a RF π-pulse tuned on the transition $|1,1\rangle - |1,0\rangle$ then implements a CNOT gate. As the frequency shifts due to interactions are usually less than 1 kHz, the gate duration is thus longer than 1 ms to avoid cross-talks with nearby transitions.

Limitations of the NMR approach. Despite its early successes, NMR quantum computing suffers from two main limitations. First, the qubit frequency in a magnetic field of 10 T is only around 50 MHz. This means that in a cryostat operating at a temperature around 1 K, the ration $\hbar\omega/(k_{\rm B}T) \sim 10^{-3}$. Thus, only a small fraction of the qubits are useful as most of them will be in a thermal mixture of $|0\rangle$ and $|1\rangle$. This reduces the signals. Second, the geometry of the molecules imposes the architecture of the couplings between qubits, thus limiting the flexibility of the approach: for each algorithm, one would have to design a specific molecule.

2 Quantum dots qubits [3]

The second approach encodes the qubits on the spin state of electrons trapped in structures called quantum dots. These structures are different from the ones we discuss for the case of single photon sources. Here, individual electrons are trapped at the local minima of the electric field produced by electrodes patterned on a semiconductor heterostructure, based for example on Silicium. This route looks promising as it benefits from the technologies developed by the semi-conductor industry, and thus is potentially highly scalable when the process is finally settled.

A typical geometry is represented in Fig. 3, with a schematic of the trapping potential. The whole structure is placed in a magnetic field around $B_0 = 1$ T, leading to

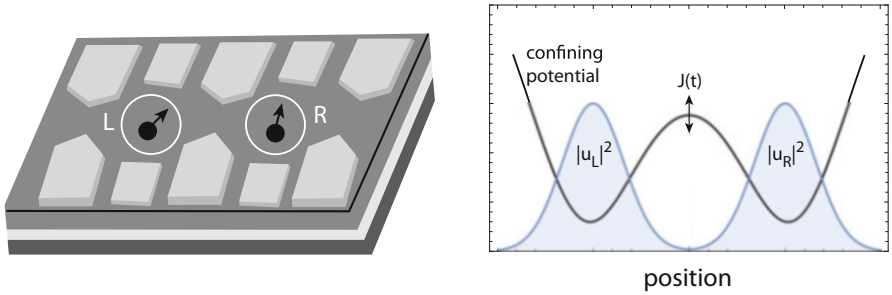

Figure 3: *Quantum dot qubits in a semiconductor heterostructure. Left: Two quantum dot qubits in top-gated potential wells. Voltages applied to the pads define the confining potential. Typical scale of devices is a dot separation of 100 nm. Right: trapping potential seen by the electrons, with the barrier which can be adjusted using the voltages applied on the electrodes (From Ref. [3]).*

a qubit frequency around 2.8 GHz. Once again, it has to operate at a cryogenic temperature to avoid thermal excitation of the qubit. The measurement of the qubit states rely on spin dependent tunneling: The height of the barrier between two nearby wells is larger than the energy separation between the qubits. The electron in state $|1\rangle$ can thus tunnel more easily towards, for example, a quantum point contact where it induces a current that can be measured. On the contrary, the tunneling is highly suppressed for electrons in $|0\rangle$. Thus the qubit state is mapped on the detection of a current or its absence.

Single qubit gates. A microwave field tuned to the transition frequency $2\mu_{\rm B}B_0/h$ induces Rabi oscillations, allowing for single-qubit gates. Demonstrations of fidelities above 99% have been achieved recently [4].

Two-qubit gates. Two electrons interact by either their magnetic moment or by the exchange interaction resulting from their electrostatic repulsion e^2/r_{12} combined to the Pauli principle. The magnetic interaction is usually completely negligible, and thus only the exchange interaction plays a role. It has the form of a Heisenberg Hamiltonian:

$$\hat{H} = \hbar J(t)\, \hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2 \ , \tag{2}$$

with the coupling constant $J(t)$ that can be controlled by shaping the trapping potential with voltages applied to the electrodes. This interaction between two electrons leads to a unitary evolution under the operator

$$\hat{U}_{\rm SWAP} = \exp\left[-i\int_0^t J(t')dt'\, \hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2\right] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}_{|00\rangle,|01\rangle,|10\rangle,|11\rangle} . \tag{3}$$

when choosing appropriately the functional form of $J(t)$.

Exercise 1. Show how to obtain the matrix from the operator in Eq. (3).

Exercise 2. (Hard) Show how the electrostatic repulsion $e^2/(4\pi\epsilon_0 r_{12})$ combined to the Pauli principle leads to the Hamiltonian (2). Hint: Consider both the external degrees of freedom of each electron in spatial wavefunctions $u_{\mathrm{L,R}}(\mathbf{r}_i)$ in the left or right well, and their total spin $S = 0, 1$. Construct properly anti-symmetrized spin dependent wavefunctions, and calculate the average repulsion energy in each of them.

Today, up to 6 electronic spin qubits have been controlled [5]. People are also using a different qubit encoding, where each qubit consists of 2 electrons in either a singlet or a triplet spin state, and reach entangling gate fidelities around 75% [6].

References

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