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Lecture 9: Introduction to decoherence and the transition to classicality

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The existence of quantum superpositions and of entanglement have been intensively, and so far successfully, tested in the microscopic world. Moreover, we have pointed out in Lecture 3 the fact that most states of a multi-partite system are entangled. It seems therefore surprising that entangled or superposition states are never observed in our macroscopic world: we never see a book half open and half closed, or a cat half dead and half alive. This was of course discussed very early on in the development of quantum physics. Yet, today, we still don't have a general answer to why this is the case. We don't even know if there exists a boundary between our classical world and the microscopic world.

In the 1980's, physicists started to realize the importance of the fact that all quantum systems are, even if very weakly, coupled to an environment with which they get entangled. This is the same idea as the one we have developed over the last lectures, here tailored to explain how the classical world could emerge from the coupling to an environment. This framework is called *decoherence*, and we will introduce its basic concepts here. This is a subtle subject, which is a consequence of the principles of quantum physics. As such, decoherence does not explain one intriguing feature of quantum physics, namely why a measurement gives a particular result on a given realization of an experiment ("collapse" of the wavefunction). But it helps understanding why classical states are special and are the ones that emerge in a measurement.

The subject is still in development. Experimentally, this is a very relevant problem, as decoherence is what, for example, makes large quantum computers so hard to build. Conceptually, one witnesses heated debates about the meaning of decoherence and the problems it solves or does not solve. For more details on this conceptual aspects, see the references at the end of this lecture notes.

1 Decoherence as a loss of coherence in large systems

We have already touched this problem in Lecture 3 when we discussed why entangled states are fragile. We rephrase here the argument using the concepts introduced in Lectures 6 and 7.

Let us consider a qubit $\alpha\,|0\rangle + \beta\,|1\rangle$ coupled to an environment that dephases it (see problem A.3 of Lecture 7). The Kraus operator associated to a dephasing is $\sqrt{\gamma}\,\hat{\sigma}_z$, and the corresponding Lindblad equation reads:

$$\frac{d\hat{\rho}}{dt} = \gamma(\hat{\sigma}_z\hat{\rho}\hat{\sigma}_z - \hat{\rho}) \ , \tag{1}$$

using $\hat{\sigma}_z^2 = 1$. This equation gives for the coherence $\rho_{01} = \langle 0|\hat{\rho}|1\rangle$, $\dot{\rho}_{01} = -2\gamma\rho_{01}$ and hence, $\rho_{01}(t) = \alpha\beta^* \, e^{-2\gamma t}$. The quantum superposition turns into a statistical (classical) mixture: $\hat{\rho} = |\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1|$. This takes a typical time $1/(2\gamma)$.

Consider instead a GHZ state of N atoms, $|\mathrm{GHZ}\rangle = (|000...\rangle + |111...\rangle)/\sqrt{2}$. The dephasing Kraus operator is now $\sqrt{\gamma}\hat{S}_z$, with $\hat{S}_z = \sum_n \hat{\sigma}_n^z$. Using $\hat{\sigma}_i\hat{\sigma}_j + \hat{\sigma}_j\hat{\sigma}_i = \delta_{i,j}$, and hence $\hat{S}_z^2 = N\,\hat{\mathbb{1}}$, the Linblad equation for the N-atom density matrix is:

$$\frac{d\hat{\rho}^{(N)}}{dt} = \gamma(\hat{S}_z\hat{\rho}^{(N)}\hat{S}_z - N\hat{\rho}^{(N)}) \ . \tag{2}$$

Therefore, the coherence $\rho_{01}^{(N)} = \langle 000...|\hat{\rho}^{(N)}|111...\rangle$ evolves as $\dot{\rho}_{01}^{(N)} = -2N\gamma\rho_{01}^{(N)}$. Hence $\rho_{01}^{(N)}(t) \sim e^{-2N\gamma t}$, and the density matrix becomes again a classical mixture

$$\hat{\rho}^{(n)} = \frac{1}{2}\,|000...\rangle\,\langle 000...| + \frac{1}{2}\,|111...\rangle\,\langle 111...| \tag{3}$$

after a time now given by $1/(2N\gamma)$. We recover the general rule that if a single qubit dephases at a rate γ by getting entangled with an environment, a superposition state of the form GHZ (often called a "cat state") dephases N times faster. As an example, take the best atomic qubit in 2024: it has a coherence time of nearly a minute. A collection of 10^6 of such qubits prepared in a GHZ state (no one has done it yet...) thus dephases in $60\,\mu$s, and one mole in 10^{-21} s! This sensitivity of entangled states to noise has been checked experimentally, for example using a chain of up to 8 trapped ions prepared in a GHZ state [5]. The results of the experiment are shown in Fig. 1.

At the end of this discussion, it looks like we have more or less solved our initial problem: we don't see superposition states of a large number of particles simply because they get entangled rapidly with the environment, thus killing the coherences, *i.e.* making the density matrix diagonal. This simple description of the decoherence process however hides many subtleties.

First, one has to be careful with the interpretation of the fact that we obtain a statistical mixture of states. For *any* density matrix one can *always* find a basis where the density matrix is diagonal, hence appearing as a statistical mixture...: but diagonalizing is a *unitary* transformation. The important point about making the density matrix diagonal by coupling the system with an environment is that it results from a *non-unitary* evolution of the system. Therefore the relevant quantity to consider is the purity of the state: decoherence transforms a pure state of a system ($\mathrm{Tr}[\hat{\rho}^2] = 1$) into a mixed state ($\mathrm{Tr}[\hat{\rho}^2] < 1$).

The second striking feature observed on the example above is the fact that the basis where the density matrix is diagonal consists of *classical* states ($|0\rangle$ or $|1\rangle$ for one qubit; $|000...\rangle$ and $|111...\rangle$ for N qubits). This is already what we know from the Schrödinger's cat tale: the two basis states into which the cat decoheres are $|\mathrm{Dead}\rangle$ and $|\mathrm{Alive}\rangle$, and

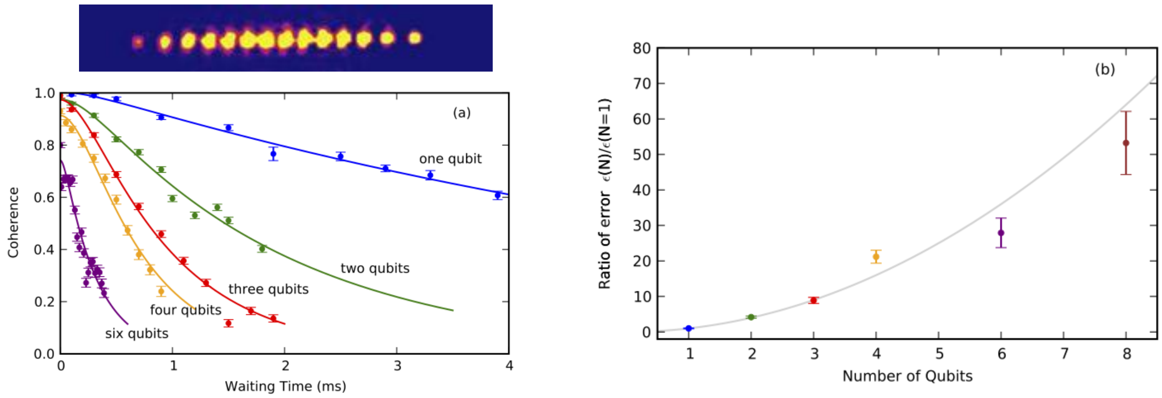

Figure 1: *(a) Evolution of the coherence $\rho_{01}^{(N)}(t)$ of a GHZ state in a chain of trapped ions, as a function of the waiting time following the preparation of the state. The various curves correspond to different ion numbers. (b) Scaling of the decoherence rate with the number of ions: here we observe a N^2 scaling instead of linear as in the text, the noise being different from pure σ_z-dephasing. Figures from [5].*

not the states $|\text{Dead}\rangle \pm |\text{Alive}\rangle$...! Why is it that the coupling to the environment seems to select the classical states as the privileged basis?

More generally, and related to the previous point, why is it that when we perform a measurement, we obtain outcomes that are classical? In a Stern and Gerlach experiment, if we send an atom in the state $|\uparrow\rangle + |\downarrow\rangle$, we do not observe a superposition of two spots... It therefore seems that there exists particular states of the measuring device (we will call them *pointer* states) that are selected in a measurement and that are classical. We will discuss this in the remaining of these notes.

2 Entanglement and "which-path" information

Let us first revisit the coupling of a qubit to an environment, and connect it to the idea of "which-path information" and the loss of coherence.

Take an initial state of a {qubit S + environment E} system of the form:

$$|\psi_{SE}(0)\rangle = (\alpha\,|0\rangle + \beta\,|1\rangle) \otimes |\mathcal{E}\rangle \ . \tag{4}$$

After some interaction time t between S and E, which we will specify later, we obtain in general an entangled state:

$$|\psi_{SE}(t)\rangle = \alpha\,|0\rangle \otimes |\mathcal{E}_0\rangle + \beta\,|1\rangle \otimes |\mathcal{E}_1\rangle \ . \tag{5}$$

Importantly, at this stage $|\mathcal{E}_0\rangle$ and $|\mathcal{E}_1\rangle$ are not necessarily orthogonal. Introducing an orthogonal basis set of the environment $\{\chi_m\}$, the density operator of the qubit is thus:

$$\begin{aligned} \hat{\rho}_S &= \mathrm{Tr}_E[|\psi_{SE}(t)\rangle\,\langle\psi_{SE}(t)|] \qquad (6)\\ &= \sum_m \langle\chi_m|\psi_{SE}(t)\rangle\langle\psi_{SE}(t)|\chi_m\rangle \\ &= |\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1| + \alpha^*\beta|1\rangle\langle 0|\langle\mathcal{E}_0|\mathcal{E}_1\rangle + \alpha\beta^*|0\rangle\langle 1|\langle\mathcal{E}_1|\mathcal{E}_0\rangle \ , \end{aligned}$$

using the completeness relation $\sum_m |\chi_m\rangle\langle\chi_m| = \hat{\mathbb{1}}$. The last line indicates that when $\langle\mathcal{E}_0|\mathcal{E}_1\rangle = 0$, *i.e.* the two states of the environment are fully distinguishable, the coherence of the state disappears. If on the contrary, the $\langle\mathcal{E}_0|\mathcal{E}_1\rangle \approx 1$, the coherence is preserved. This is an example of "which-path" detection of the state of the qubit by the state of the environment (even if we don't measure it...).

Generally speaking, $\langle\mathcal{E}_0(t)|\mathcal{E}_1(t)\rangle \to 0$ in a time scale which decreases rapidly with the size of the environment. To see that, let us introduce a model proposed by Zurek, that will be very useful when discussing the emergence of classical states in the next section. The environment consists of an ensemble of N qubits k coupled to the qubit S by an Ising-like Hamiltonian:

$$H_{SE} = \sum_{k=1}^{N} \hbar g_k \hat{\sigma}^z \otimes \hat{\sigma}_k^z \; . \tag{7}$$

Starting from the state of $S + E$:

$$|\psi_{SE}(0)\rangle = (\alpha\,|0\rangle + \beta\,|1\rangle) \otimes \prod_{k=1}^{N} (\alpha_k\,|0\rangle_k + \beta_k\,|1\rangle_k) \; , \tag{8}$$

the state at a later time becomes

$$|\psi_{SE}(t)\rangle = \exp[-i\frac{H_{SE}}{\hbar}t]\,|\psi_{SE}(0)\rangle \tag{9}$$

$$= \alpha\,|0\rangle \otimes |\mathcal{E}_0(t)\rangle + \beta\,|1\rangle \otimes |\mathcal{E}_1(t)\rangle \tag{10}$$

with the two states of the environment being:

$$|\mathcal{E}_0(t)\rangle = \prod_{k=1}^{N} (\alpha_k e^{-ig_k t}\,|0\rangle_k + \beta_k e^{ig_k t}\,|1\rangle_k) \; , \; |\mathcal{E}_1(t)\rangle = \prod_{k=1}^{N} (\alpha_k e^{ig_k t}\,|0\rangle_k + \beta_k e^{-ig_k t}\,|1\rangle_k) \; . \tag{11}$$

The overlap between these two states thus evolves as:

$$\langle\mathcal{E}_0(t)|\mathcal{E}_1(t)\rangle = \prod_{k=1}^{N} (|\alpha_k|^2 e^{2ig_k t} + |\beta_k|^2 e^{-2ig_k t}) \; . \tag{12}$$

To get an idea of how quickly this terms goes to 0, let us take $\alpha_k = \beta_k = 1/\sqrt{2}$, and $g_k = g$ for all k. Then

$$\langle\mathcal{E}_0(t)|\mathcal{E}_1(t)\rangle = \cos(2gt)^N \approx e^{-2Ng^2t^2} \quad \text{for} \quad t \to 0 \; . \tag{13}$$

Hence in a time $T_c \sim 1/(2\sqrt{N}g)$, the coherence between the qubit states $|0\rangle$ and $|1\rangle$ disappear, and the larger the reservoir the faster the decoherence. The recurrence time, *i.e.* the time after which the overlap goes back to 1, is $T_{\rm r} = \pi/(2g)$: it can be extremely long if the coupling $S - E$, g, is small. If, moreover, the g_k's are random, and so are the α_k's and β_k's, the recurrence time can easily exceed the age of the universe for an environment consisting of a macroscopic number of qubits...

Interestingly, with the choice of coupling Hamiltonian (7), had we taken $\alpha_k = 1$ (thus $\beta_k = 0$), we would have obtained $|\langle\mathcal{E}_0(t)|\mathcal{E}_1(t)\rangle| = 1$ at all time: the coherence would have never disappeared!

3 Measurement and classicality: pointer states

We now have everything in place to understand a bit more in detail the measurement process and why classical states emerge from it.

We consider again a system S, a qubit, coupled now to a measuring device M (also called a *meter*). The goal of a measurement is to acquire information about the system by looking at the measuring device: the measurement process thus require an interaction between S and M. Initially, $S-M$ is prepared in the state:

$$|\psi_{SM}(0)\rangle = (\alpha\,|0\rangle + \beta\,|1\rangle) \otimes |\chi\rangle_M \ . \tag{14}$$

During the measuring process, S and M get entangled:

$$|\psi_{SM}(t)\rangle = \alpha\,|0\rangle \otimes |\chi_0\rangle_M + \beta\,|1\rangle \otimes |\chi_1\rangle_M \ . \tag{15}$$

For the measurement to be successful, we should be able to distinguish between the two states of the measuring device, in order to obtain a "which-path" information. As we discussed in Sec.2, this require $\langle\chi_0|\chi_1\rangle = 0$. We will call this two states of the meter the *pointer states*: they could be two positions of the needle of a voltmeter, two distinct values of a photocurrent... In any case, we know from our daily experience that these pointer states are "classical".

Tracing over the pointer states of the meter, the density matrix of the qubit is thus $\hat{\rho}_S = |\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1|$. Everything seems to look good: we obtain a statistical mixture of $|0\rangle$ and $|1\rangle$. But let us now introduce the states $|\pm\rangle_S = (|0\rangle \pm |1\rangle)/\sqrt{2}$ and $|\chi_\pm\rangle_M = (\alpha\,|\chi_0\rangle_M \pm \beta\,|\chi_1\rangle_M)$. The state (15) can be rewritten:

$$|\psi_{SM}(t)\rangle = \frac{1}{\sqrt{2}}\,|+\rangle_S \otimes |\chi_+\rangle_M + \frac{1}{\sqrt{2}}\,|-\rangle_S \otimes |\chi_-\rangle_M \ . \tag{16}$$

Taking for example $\alpha = \beta = 1/\sqrt{2}$, we find $\langle\chi_+|\chi_-\rangle = 0$ and the density matrix of the qubit is diagonal again in the $|\pm\rangle_S$ basis..., and that would be true for any other choice of orthogonal basis! This problem is called the *basis ambiguity*: using again the language of a Stern-Gerlach experiment this means that for a spin in a superposition of $|\uparrow\rangle + |\downarrow\rangle$ we should be able to observe a superposition of two spots. In the same way, an old-fashion voltmeter with a needle could be seen pointing in two directions at the same time...

To solve this basis ambiguity, we must recognize the fact that the measuring device itself is coupled to an environment. This may not be totally surprising: after all, if we imagine a voltmeter with a needle pointing Left when a qubit is in state $|0\rangle$ and Right when it is in $|1\rangle$, and if we assume that the system $S-M$ is isolated, S and M will exchange coherently their energy. The needle will oscillate between its Left and Right positions while the qubit oscillates between $|0\rangle$ and $|1\rangle$... Something must damp the motion of the needle so that it points in a direction from which you can extract some information about the state of the qubit. We therefore need to consider in the measurement process three partners: the system E, the measuring device M, and an environment E coupled to the meter M.

Initially, the state of $S-M-E$ is:

$$|\psi_{SME}(0)\rangle = (\alpha\,|0\rangle + \beta\,|1\rangle)_S \otimes |\chi\rangle_M \otimes |\mathcal{E}\rangle_E \ . \tag{17}$$

We now decompose the measurement process in two steps. In the first one, called a *pre-measurement*, S and M get entangled:

$$|\psi_{SME}\rangle = (\alpha\,|0\rangle_S \otimes |\chi_0\rangle_M + \beta\,|1\rangle_S \otimes |\chi_1\rangle_M) \otimes |\mathcal{E}\rangle_E \ . \tag{18}$$

In a second step, the interaction between the environment and M leads to a state:

$$|\psi_{SME}\rangle = \alpha\,|0\rangle_S \otimes |\chi_0\rangle_M \otimes |\mathcal{E}_0\rangle_E + \beta\,|1\rangle_S \otimes |\chi_1\rangle_M \otimes |\mathcal{E}_1\rangle_E \ . \tag{19}$$

We can now calculate the reduced density matrix of the $S-M$ system:

$$\begin{aligned} \hat{\rho}_{SM} &= \mathrm{Tr}_E[|\psi_{SME}\rangle\,\langle\psi_{SME}|] \\ &= |\alpha|^2|0\rangle\langle 0|_S \otimes |\chi_0\rangle\langle\chi_0|_M + |\beta|^2|1\rangle\langle 1|_S \otimes |\chi_1\rangle\langle\chi_1|_M \\ &\quad +\alpha\beta^*\langle\mathcal{E}_1|\mathcal{E}_0\rangle|0\rangle\langle 1|_S\langle\otimes|\chi_0\rangle\langle\chi_1|_M + \text{h.c.}\ . \end{aligned} \tag{20}$$

Again, the coherence depends on the overlap between two states of the environment.

To move one, we have to specify a particular form of the $M-E$ coupling. Here again we will assume that the environment consists of N qubits k, coupled to the meter M. Let us take an Ising-like coupling analogous to the one of Eq. (7):

$$H_{ME} = (|\chi_0\rangle\langle\chi_0| - |\chi_1\rangle\langle\chi_1|)_M \otimes \sum_{k=1}^{N} \hbar g_k \hat{\sigma}_k^z \ . \tag{21}$$

The calculation is the same as the one leading to Eq. (12). We thus conclude that for a large environment ($N \gg 1$), $\langle\mathcal{E}_0|\mathcal{E}_1\rangle \to 0$ in a very short amount of time, and therefore the coherence between the meter and the qubit disappears: we are left with a classical mixture where the qubit is in $|0\rangle$, the meter being in $|\chi_0\rangle$ or the qubit is in $|1\rangle$ with the meter in $|\chi_1\rangle$. Hence the coupling to the environment has selected the pointer states (classical states) as the ones that lead to a mixed states between them and the state of the qubit.

What is the magic? There is none. We have simply taken for the $M-E$ coupling Hamiltonian a form diagonal in the pointer states: starting from an initial factorized state $|\chi_0\rangle_M \otimes |\mathcal{E}\rangle$ or $|\chi_1\rangle_M \otimes |\mathcal{E}\rangle$, we obtain

$$\exp[-i\frac{H_{ME}}{\hbar}t]\,|\chi_{0,1}\rangle_M \otimes |\mathcal{E}\rangle = |\chi_{0,1}\rangle_M \otimes |\mathcal{E}_{0,1}(t)\rangle \ . \tag{22}$$

The coupling does not lead to any entanglement between the meter and the environment! Things would have been different had we started from the states $|\chi_\pm\rangle_M \otimes |\mathcal{E}\rangle$ (for example a superposition of two spots in the Stern and Gerlach experiment, or the state $|\text{Dead}\rangle \pm |\text{Alive}\rangle$ of the Cat). Then, the coupling to the environment would have produced an entangled state of the meter and the environment:

$$\exp[-i\frac{H_{ME}}{\hbar}t]\,|\chi_\pm\rangle_M \otimes |\mathcal{E}\rangle = \frac{1}{\sqrt{2}}(|\chi_0\rangle_M \otimes |\mathcal{E}_0(t)\rangle \pm |\chi_1\rangle_M \otimes |\mathcal{E}_1(t)\rangle)\ . \tag{23}$$

And we know that such entangled state decoheres very rapidly, as $\langle\mathcal{E}_0(t)|\mathcal{E}_1(t)\rangle \to 0$!

One may argue that the reason why we find that the pointer (classical) states constitute the privileged basis selected by the environment results from the particular form of the meter-environment Hamiltonian we have chosen. This is of course true... But we have learned something very important: the pointer states are the ones that do not get entangled with the environment, *i.e.* that are robust to decoherence. Now, in a given situation for which we know what are the pointer states, simply because they are the states of our measuring apparatus in a given experiment, we can construct a coupling H_{ME} adapted to the situation and then see whether it can correspond to a physically possible meter-environment coupling...

There doesn't seem to exist general rule to find H_{ME}. However, one sufficient condition is too look for Hamiltonian that commute with all the projectors $\hat{P}_m^{(M)} = |\chi_m\rangle\langle\chi_m|$ of the pointer states $\{|\chi_m\rangle\}$ of the meter:

$$[H_{ME}, \hat{P}_m^{(M)}] = 0 \quad \forall\ m\ . \tag{24}$$

As any observable $\hat{O}_M$ of the meter is of the form $\hat{O}_M = \sum_m o_m \hat{P}_m^{(M)}$, the condition (24) is equivalent to $[H_{ME}, \hat{O}_M] = 0$. As we saw in Lecture 5, this is the condition of a quantum non-demolition measurement of the meter by the environment!

4 Coherent state as pointer states and the decoherence of Schrödinger cats

Let us know reverse the problem and assume that we have guessed from practical considerations a given meter-environment coupling. Can we find the corresponding pointer states? Here again there is no general rule and the task has been successfully achieved only for a limited number of models. A general approach consists in studying the time evolution of a large set of pure states and select those that remain pure, or at least that get weakly entangled with the environment. This method is called the *predictability sieve*. In practice, one solves the Lindblad equation of the model (most often numerically) for a variety of initial pure states. One then studies the time evolution of the purity $\mathrm{Tr}[\hat{\rho}^2]$ or the von Neumann entropy, and post-selects the states for which these quantities do not evolve or evolve the least.

A special case where one can explicitly find the pointer states is the damped harmonic oscillator. It can be a real physical oscillator as a particle in a quadratic potential or a nano-resonator, but also a mode of a quantized electromagnetic field in a cavity (see Lecture 2). The damping comes from a coupling to an environment: for the cavity this is the loss through the mirror of the cavites; for a mechanical oscillator it can be some friction. Let us define the damping rate κ as the rate of energy loss, so that $d\langle E\rangle/dt = -\kappa\langle E\rangle$. As we saw at the end of Lecture 6, assuming that the environment is in its ground state, only one quantum channel contributes to the damping, with the jump operator $\hat{L}_- = \sqrt{\kappa}\hat{a}$. The damping part of the Lindblad equation is:

$$\frac{d\hat{\rho}}{dt} = \mathcal{L}[\hat{\rho}] = \frac{\kappa}{2}\left(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{\rho}\hat{a}^\dagger\hat{a} - \hat{a}^\dagger\hat{a}\hat{\rho}\right)\ . \tag{25}$$

We want to find the pointer states associated to this equation. It turns out that the coherent states:

$$|\alpha(t)\rangle = e^{-|\alpha(t)|^2/2} \sum_{n=0}^{\infty} \frac{\alpha(t)^n}{\sqrt{n!}} |n\rangle \ , \tag{26}$$

with $\alpha(t) = \alpha \exp[-\kappa t/2]$ are such states. This can be checked directly by showing that the density matrix $\hat{\rho}(t) = |\alpha(t)\rangle\langle\alpha(t)|$ is solution of the equation (25). To do so, write

$$\begin{aligned} \frac{d\,|\alpha(t)\rangle}{dt} &= -\frac{1}{2}\frac{d|\alpha(t)|^2}{dt}|\alpha\rangle + e^{-|\alpha(t)|^2/2}\frac{d\alpha(t)}{dt}\sum_n \frac{n\alpha^{n-1}(t)}{\sqrt{n!}}|n\rangle \\ &= \frac{\kappa}{2}|\alpha(t)|^2|\alpha\rangle - \frac{\kappa}{2}\hat{a}^\dagger\hat{a}|\alpha\rangle \ . \end{aligned} \tag{27}$$

This leads to

$$\frac{d|\alpha(t)\rangle\langle\alpha(t)|}{dt} = \kappa|\alpha(t)|^2|\alpha(t)\rangle\langle\alpha(t)| - \frac{\kappa}{2}\hat{a}^\dagger\hat{a}|\alpha(t)\rangle\langle\alpha(t)| - \frac{\kappa}{2}|\alpha(t)\rangle\langle\alpha(t)|\hat{a}^\dagger\hat{a} \ . \tag{28}$$

As $|\alpha|^2|\alpha\rangle\langle\alpha| = \alpha|\alpha\rangle\langle\alpha|\alpha^* = \hat{a}|\alpha\rangle\langle\alpha|\hat{a}^\dagger$, we obtain the Lindblad equation (25) applied to $\hat{\rho}(t) = |\alpha(t)\rangle\langle\alpha(t)|$. This shows that a coherent state remains coherent during the damping, indicating that it does not get entangled with the environment: $|\alpha\rangle_S \otimes |\mathcal{E}\rangle_E$ evolves into $|\alpha(t)\rangle_S \otimes |\mathcal{E}(t)\rangle_E$. This is consistent with the idea that coherent states are the quantum states closest to classical states. However, this does not mean that a coherent state is unaffected by the environment: the coupling with it induces an exponential relaxation of the normal variable $\alpha(t)$ and, thus of the oscillations.

A superposition of pointer states is however in general *not* a pointer state. Let us consider for example the superposition of two coherent states $|\alpha_+\rangle$ and $|\alpha_-\rangle$

$$|\psi(0)\rangle = \frac{|\alpha_+\rangle + |\alpha_-\rangle}{\sqrt{2\big[1 + \mathrm{Re}(\langle\alpha_+|\alpha_-\rangle)\big]}} \ , \tag{29}$$

For $|\alpha_\pm| \gg 1$ and $|\alpha_+ - \alpha_-| \gg 1$, it is a Schrödinger cat state, *i.e.* a superposition of *distinct* macroscopic states. The correction to the normalization factor is unimportant here because (i) $|\langle\alpha_+|\alpha_-\rangle| = \exp(-|\alpha_+ - \alpha_-|^2/2) \ll 1$ and, (ii) the Lindblad equation preserves the trace, *i.e.* the normalization. We can thus use the (unnormalized) initial density matrix

$$\tilde{\rho}(0) = \frac{1}{2}\Big[|\alpha_+\rangle\langle\alpha_+| + |\alpha_-\rangle\langle\alpha_-| + |\alpha_+\rangle\langle\alpha_-| + |\alpha_-\rangle\langle\alpha_+|\Big] \ . \tag{30}$$

The states $|\alpha_+\rangle$ and $|\alpha_-\rangle$ being quasi-orthogonal (for $|\alpha_+ - \alpha_-| \ll 1$), the first two terms can be interpreted as "populations" and the last two as 'coherences" in the quasi-basis made of the states $|\alpha_\pm\rangle$. The Lindblad equation being linear, each term evolves independently of the others.

$$|\alpha_\pm\rangle\langle\alpha_\pm| \to |\alpha_\pm(t)\rangle\langle\alpha_\pm(t)| \qquad \text{with} \qquad \alpha_\pm(t) = \alpha_\pm e^{-\kappa t/2} \ , \tag{31}$$

since each coherent state is a solution of the Lindblad equation. In contrast, the coherence terms $|\alpha_+(t)\rangle\langle\alpha_-(t)|$ and $|\alpha_-(t)\rangle\langle\alpha_+(t)|$ are not solutions of the Lindblad equation. A calculation analogous to the one leading to Eq. (28) gives

$$\frac{d|\alpha_+\rangle\langle\alpha_-|}{dt} = \mathcal{L}\big(|\alpha_+\rangle\langle\alpha_-|\big) + \frac{\kappa}{2}\left(|\alpha_+|^2 + |\alpha_-|^2 - 2\alpha_+\alpha_-^*\right)|\alpha_+\rangle\langle\alpha_-| \ , \tag{32}$$

where $\mathcal{L}$ is the Lindbladian defined at Eq. (25). The solution of the Lindblad equation for this coherence term is the evolution $|\alpha_+\rangle\langle\alpha_-| \to e^{-\Phi(t)}|\alpha_+(t)\rangle\langle\alpha_-(t)|$, with

$$\Phi(t) = \frac{1}{2}\Big[|\alpha_+|^2 + |\alpha_-|^2 - 2\alpha_+\alpha_-^*\Big]\left(1 - e^{-\kappa t}\right) \ . \tag{33}$$

Taking $\alpha_+ = -\alpha_- = \alpha$ (real and positive), we find $\Phi(t) = 2\alpha^2\left(1 - e^{-\kappa t}\right)$. The solution of the Lindblad equation with a superposition of coherent states as an initial state, Eq. (29), is thus

$$\tilde{\rho}(t) = \frac{1}{2}\Big[|\alpha(t)\rangle\langle\alpha(t)| + |-\alpha(t)\rangle\langle-\alpha(t)| + e^{-\Phi(t)}\Big(|\alpha(t)\rangle\langle-\alpha(t)| + |-\alpha(t)\rangle\langle\alpha(t)|\Big)\Big] \ . \tag{34}$$

This dynamics is characterized by three different time scales. First, the population terms decay towards the ground state $|0\rangle\langle 0|$ in a time $\tau_{\mathrm{decay}} = 1/\kappa$, as in the case of a single coherent state. Second, the two coherent states start to merge after a typical time such that $|\alpha_+(t) - \alpha_-(t)| \simeq 1$, leading to $\tau_{\mathrm{merge}} = \ln(2\alpha)/\kappa$. This time is larger than τ_{decay} for $\alpha \gg 1$. Third, the coherence terms decrease much faster, as their amplitude is:

$$\left|e^{-\Phi(t)}\right| = \exp\left[-2\alpha^2\left(1 - e^{-\kappa t}\right)\right] \ . \tag{35}$$

At short times, $\Phi(t) \approx 2\alpha^2\kappa t$, and the amplitude of the coherence terms decays exponentially. At long time, $t \gg 1/\kappa$, it saturates to $\exp\left(-2\alpha^2\right) \ll 1$. The coherence terms thus become negligible in a time $\tau_{\mathrm{decoh}} = 1/(2\alpha\kappa) \ll 1/\kappa$. We recover here again the usual conclusion that the larger a cat, the faster it decoheres towards a statistical mixture of its two classical components. All these predictions have been tested experimentally using a setup similar to the one we discussed in Lecture 5 about QND measurements [6]: a coherent state of an electromagnetic field is prepared in a superconducting microwave cavity. Sending a Rydberg atom with resonant frequency different from the cavity realizes a dispersive coupling between atoms and field and prepares a Schrödinger cat of the form $|\alpha e^{i\phi}\rangle + |\alpha e^{-i\phi}\rangle$. After some time a second atom is sent through the cavity to probe the field. In this way, one extracts a quantity proportional to $\mathrm{Re}[e^{-\Phi(t)}]$. The results of the experiment are shown in Fig. 2, together with the predictions of Eq. (33).

5 Summary and conclusion

The decoherence program that we have outlined above on specific examples is fully contained into the formalism of quantum physics: it thus does not bring any new ideas. It simply emphasizes the consequences of considering that a quantum system is necessarily coupled to an outside world (as long as we don't consider for the system the universe as a whole...): the most important one is the entanglement between the system and the environment that, from the point of view of the system, looks like a loss of coherence in a very short time set by the size of the environment. The formalism also shows that "large" quantum systems decohere faster than "small" ones.

The second important aspect that decoherence highlights is the fact that classical states are the ones stable against decoherence: this is almost a tautological statement, as precisely the state that we observe in our daily life are the ones that survived decoherence... There thus seems to exist an environment induced selection (one speaks

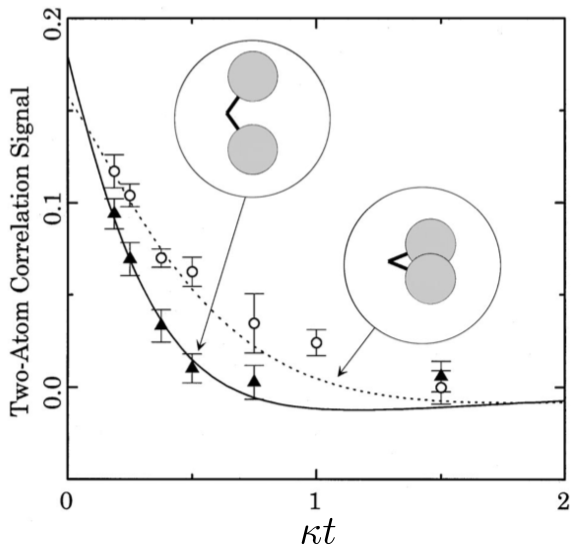

Figure 2: *Decoherence of a Schrödinger cat state of the form* $|\alpha e^{i\phi}\rangle + |\alpha e^{-i\phi}\rangle$*. The cat is prepared in a microwave superconducting cavity with* $1/\kappa = 160\,\mu s$*, containing* $|\alpha|^2 \approx 3.5$ *photons. The state of the field is measured by sending Rydberg atoms through the cavity using a method similar to the one explained in Lecture 5. The figure shows a signal proportional to the coherence between the two components of the cat* $\mathrm{Re}[e^{-\Phi(t)}]$*. The solid line is the prediction for* $\Phi(t)$ *using Eq. (33). Figures from [6].*

of *einselection*) of stable preferred states, the classical ones. Sometimes one speaks of "Quantum Darwinism"...

Nevertheless, decoherence does not explain why, when we have our diagonal density matrix $S-M$, we obtain only one of the possible results, and why this one...

References

[1] W. H. Zurek, "Decoherence and the transition from quantum to classical", Physics Today **44**, 36 (1991).

[2] W. H. Zurek, "Decoherence, einselection and the quantum origins of the classical", Rev. Mod. Phys. **75**, 715 (2003).

[3] M. Schlosshauer, "Decoherence, the measurement problem, and interpretations of quantum mechanics", Rev. Mod. Phys. **76**, 715 (2004).

[4] Erich Joos, "Decoherence: An Introduction", Physics and Philosophy (2007).

[5] T. Monz, *et al.* "14-Qubit Entanglement: Creation and Coherence", Phys. Rev. Lett. **106**, 130506 (2011).

[6] M. Brune, *et al.* "Observing the Progressive Decoherence of the 'Meter' in a Quantum Measurement", Phys. Rev. Lett. **77**, 4887 (1996).