Physics of Quantum Information

Lecture 5 – Quantum measurement

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Measurements in Quantum Physics

Measurements are fundamentally quantum processes, different from classical physics

Well established operational formalism, compatible with any experiment so far

Debates and philosophical questions about its interpretation and in particular the role of the observer

→ Focus on a concrete discusion of measurements

→ Yet, standard measurement theory is far incomplete

Many subtle concepts

Intrinsically probabilistic and wavepacket collapse

Projection valued measurements (PVM), positive operator valued measurements (POVM)

Quantum nondemolition (QND) measurements

...

→ A measurement primarily serves to acquire information about the state of the system

Outlook

1. Projective measurements (PVM)

   1.1 Projective measurement of a pure state (reminder)

   1.2 Projective measurement of a mixed state

2. Quantum nondemolition (QND) measurements

   2.1 Motivation

   ~~2.2 QND measurement : Formal definition~~

   2.3 QND measurement in practice

   2.4 QND in cavity quantum electrodynamics

3. Generalized measurements (POVM)

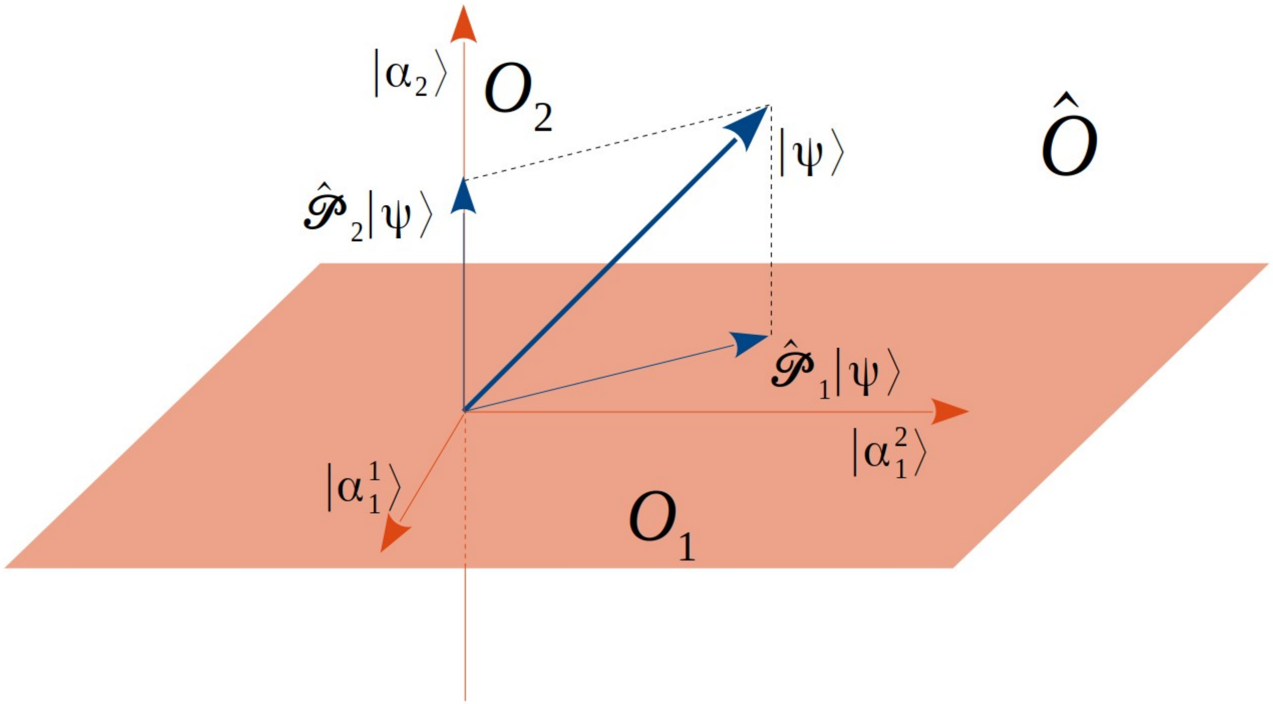
   3.1 Describing a generalized measure from the unitary evolution of a larger system

   3.2 Generalized measurement of a pure state

   3.3 Generalized measurement of a mixed state

   3.4 Use cases of generalized measurements → see homework

Projection-Valued Measurements (PVM)



Standard measurements are described by projectors

$$\mathscr{P}_j = \sum_\nu |\alpha_j^\nu\rangle\langle\alpha_j^\nu|$$

	Pure state	Mixed state
Probability	$P_j = \lvert\hat{\mathscr{P}}_j\lvert\Psi\rangle\rvert^2$	$P_j = \mathrm{Tr}\left(\hat{\mathscr{P}}_j\hat{\rho}\right)$
Conditional after-measurement state	$\lvert\Psi_{\lvert j}'\rangle = \dfrac{\hat{\mathscr{P}}_j\lvert\Psi\rangle}{\lvert\hat{\mathscr{P}}_j\lvert\Psi\rangle\rvert}$	$\hat{\rho}_{\lvert j}' = \dfrac{\hat{\mathscr{P}}_j\hat{\rho}\hat{\mathscr{P}}_j}{\mathrm{Tr}\left(\hat{\mathscr{P}}_j\hat{\rho}\right)}$
State after an unread measurement	$\hat{\rho}' = \sum_j \hat{\mathscr{P}}_j\lvert\Psi\rangle\langle\Psi\rvert\hat{\mathscr{P}}_j$	$\hat{\rho}' = \sum_j \hat{\mathscr{P}}_j\hat{\rho}\hat{\mathscr{P}}_j$

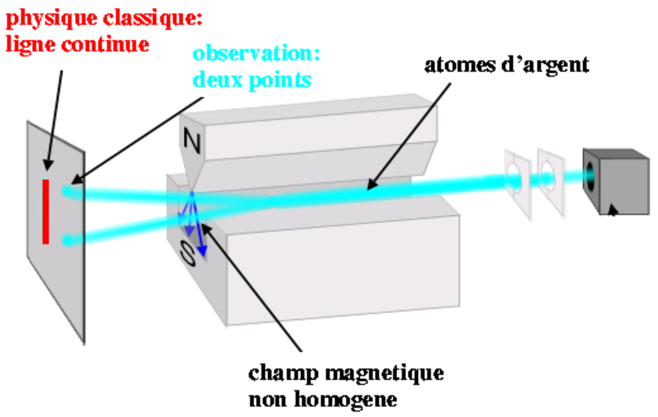
Standard (Demolishing) Measurements

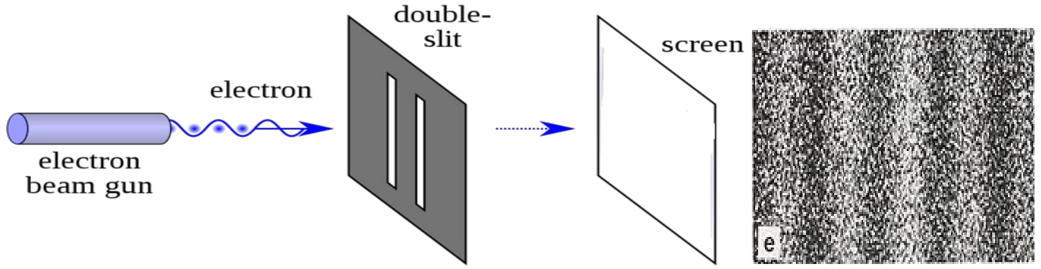
Measurements in quantum physics have two main consequences

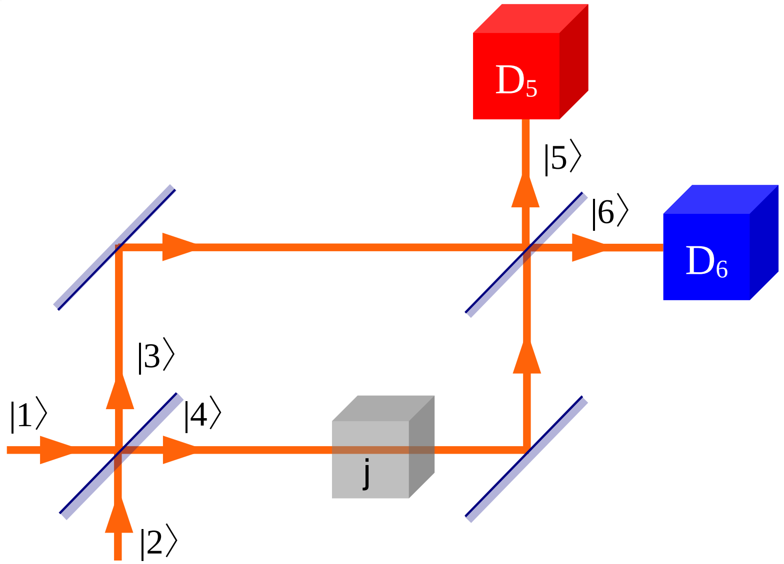
Wavepacket collapse : In general the state of the system (be it pure or mixed) changes after the measurement

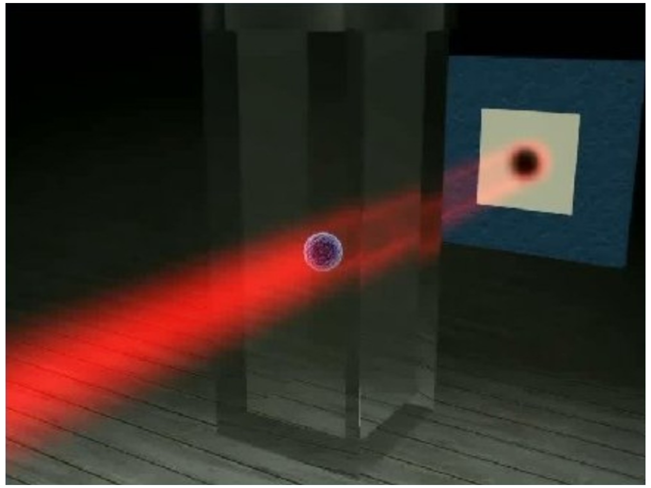
→ To our knowledge, there is strictly nothing we can do against this !

In usual cases, the system is demolished (no longer exists) after the measurement









Standard (Demolishing) Measurements

Drawbacks of demolishing measurement

i) One cannot check the wavepacket collapse rule

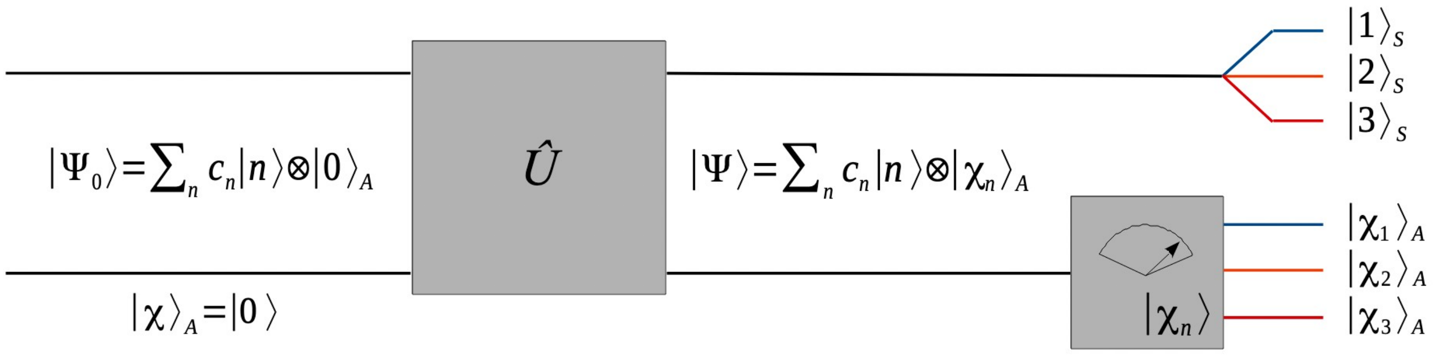
ii) Demolishing measurement impedes state preparation by measurement

iii) One cannot track the time evolution of a quantum system (eg Zeno effect, life and death of a photon, …)

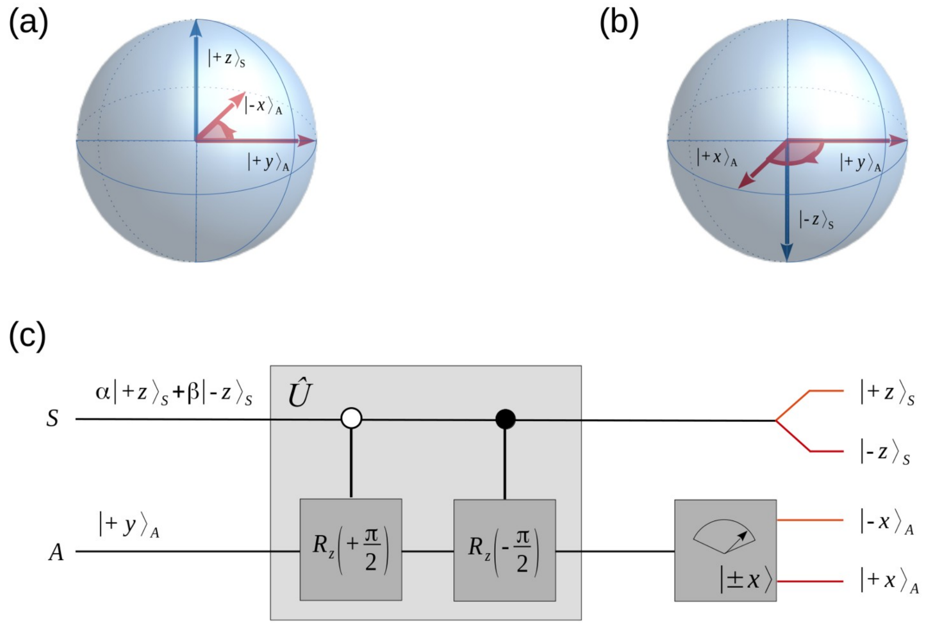
→ In contrast to wavepacket collapse, physical demolition can be overcome !

Quantum Non Demolition Measurements

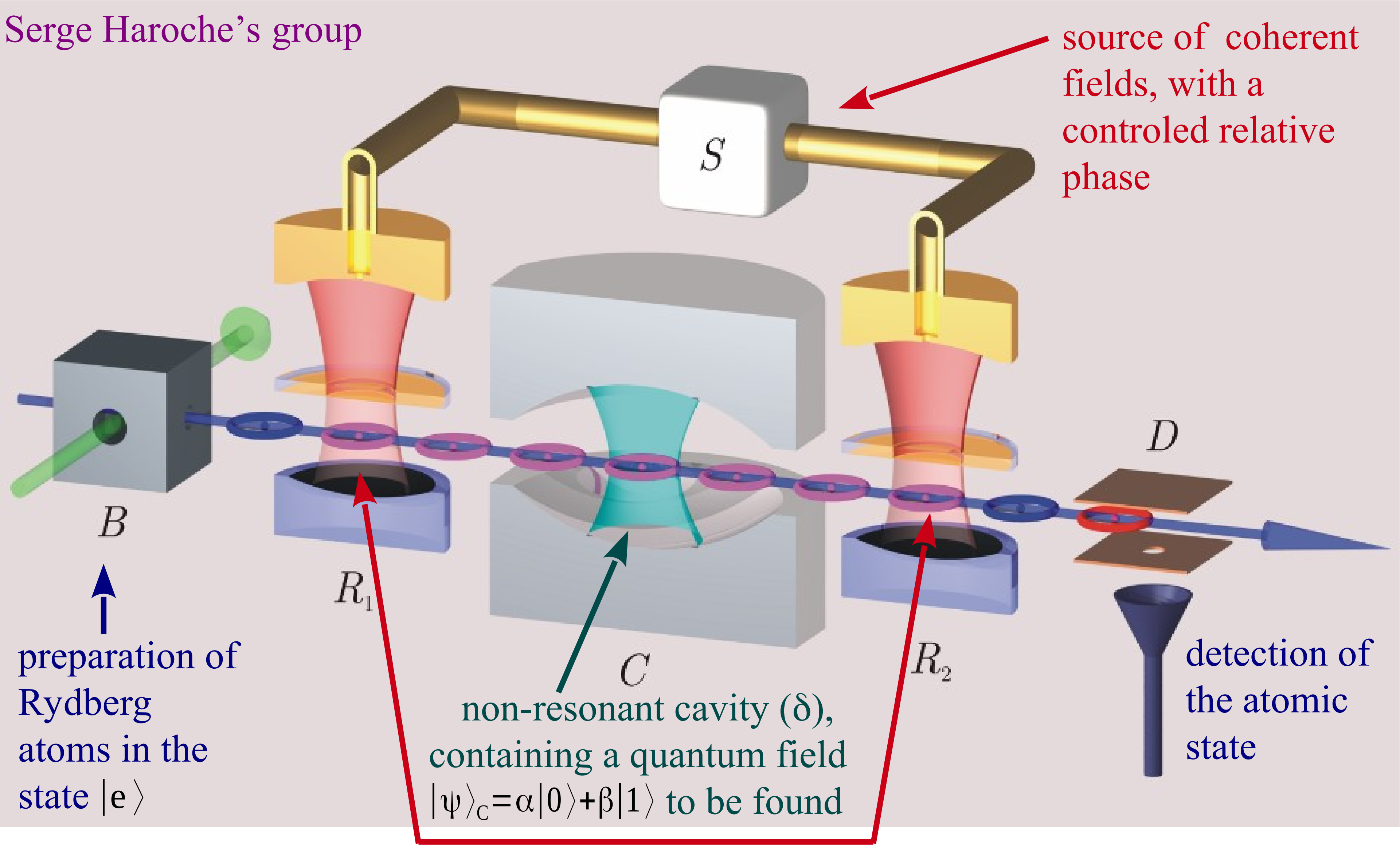
General principle of a QND measurement



A practical example for a qubit



Quantum Non Demolition (QND) Measurements in a Cavity

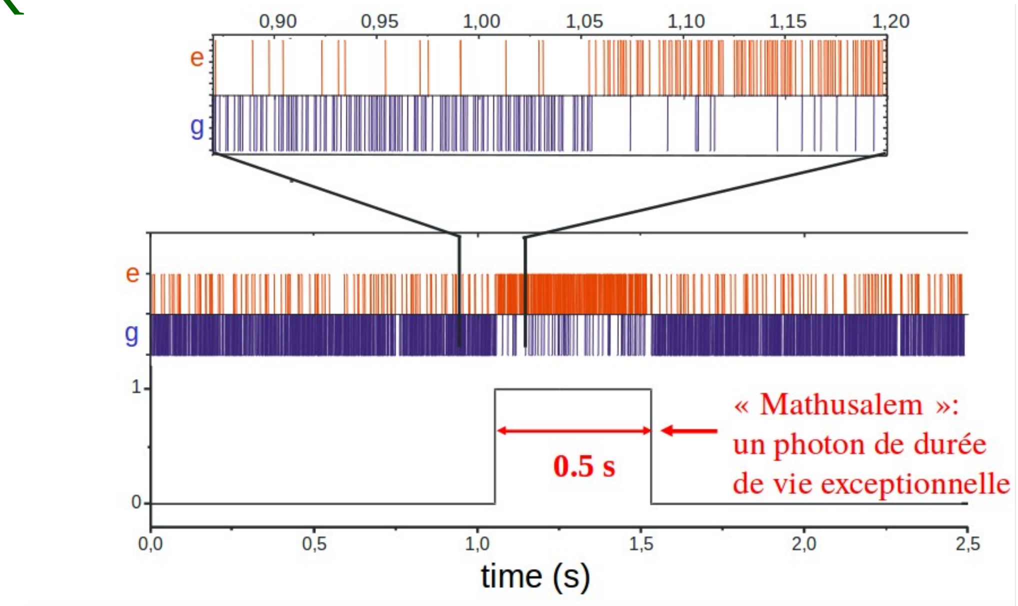


resonant cavities, containing quasi-classical fields; manipulation of the atomic state ($\pi/2$ pulses)

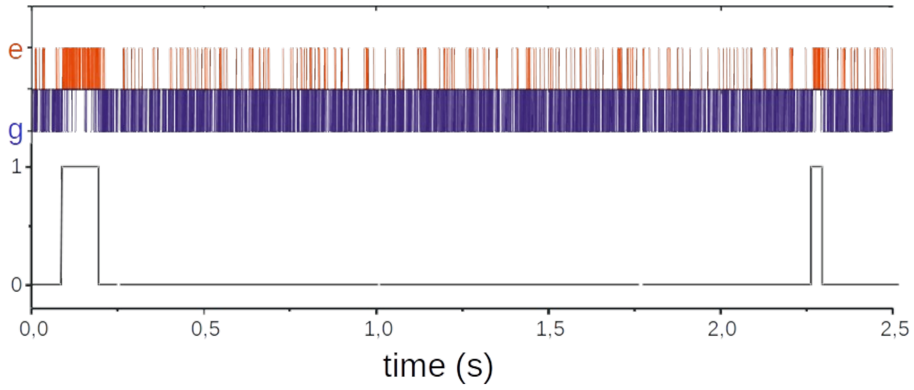
- 10% of the atoms are detected
- 200 atoms by photon lifetime in the cavity (~130 ms)

Life and Death of a Photon

Mesurement at T=0.8K



Mesurement at T=1.5K



Gleyzes *et al.*, *Nature* **446**, 297 (2007)

Positive Operator Valued Measurements (POVM)

Many important measurements are not described by PVMs but by POVMs

Measurement of the state of an atom by ionisation, $\hat{M}_{\mathrm{g/e}} = |\mathrm{ion}\rangle\langle \mathrm{g/e}|$

Photo-detection (see course on Quantum Optics), $\hat{M} = \hat{E}(\vec{r}, t)$

Photon counter, $\hat{M}_n = |0\rangle\langle n|$

→ Imperfect measurements, but sometimes more powerful (see Homework)

Generalized measurements are described by non-projection operators

	Pure state	Mixed state
Probability	$P_m = \lvert \hat{M}_m \lvert\psi\rangle \rvert^2$	$P_m = \mathrm{Tr}\left(\hat{M}_m \hat{\rho} \hat{M}_m^\dagger\right)$
Conditional after-measurement state	$\lvert\Psi_{\lvert j}{}'\rangle = \dfrac{\hat{M}_m \lvert\psi\rangle}{\lvert \hat{M}_m \lvert\psi\rangle \rvert}$	$\hat{\rho}_{\lvert m}{}' = \dfrac{\hat{M}_m \hat{\rho} \hat{M}_m^\dagger}{\mathrm{Tr}\left(\hat{M}_m \hat{\rho} \hat{M}_m^\dagger\right)}$
State after an unread measurement	$\hat{\rho}' = \sum_m \hat{M}_m \lvert\psi\rangle\langle\psi\rvert \hat{M}_m^\dagger$	$\hat{\rho}' = \sum_m \hat{M}_m \hat{\rho} \hat{M}_m^\dagger$

The Kraus operators in POVMs play the same role as the projectors in PVMs. In general, they are less efficient but more numerous !